SINGULAR TROUBLES WITH SINGLETON SOCRATES

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1 INTRODUCTION

Some facts obtain in virtue of other facts, which may then be said to *ground* the former. Many important metaphysical questions concern matters of ground: whether normative facts are grounded by non-normative ones, whether mental facts are grounded by physical ones, and so on. In order that debates on such questions can proceed in a fruitful and rigorous fashion, it is desirable that we have an appropriate formal framework available, within which the various competing positions can be articulated and their consequences drawn out. As part of this, it is desirable that we have an adequate semantics for statements of ground. Ideally, this will help clarify and structure debates on metaphysical ground in a similar way in which possible worlds semantics does this for debates about metaphysical necessity.

A promising candidate for this role is the *truthmaker semantics* for ground, as developed by Kit Fine in several recent publications (2012a; 2012b; 2017b). The basic idea of this approach is to model ground as a form of *entailment*, which is however characterized by an inclusion relation not between the sets of *worlds* in which premises and conclusion are true, but between the sets of *situations* or *states* that *make* premises and conclusion true. The relationship of truthmaking here may be seen as a (partial) semantic correlate of the relationship of ground that is expressed in the object language. Roughly, a state is taken to make a proposition true iff the state's obtaining would ground the truth of the proposition. Correspondingly, the truthmaking relation is taken to share several distinctive logico-structural features of grounding. In particular, the truthmaker must *necessitate* what it makes true, much as grounds are standardly taken to necessitate what they ground, it must be *relevant* to what it makes true, much as grounds are to what they ground, and like ground, the truthmaking relation is consequently *non-monotonic*.

In the first part of the paper, I present a problem for the truthmaker semantics for ground. The problem consists in the fact that there seem to be possible *grounding structures* which are ruled out by the logic of ground that we obtain under the truthmaker account. Specifically, as I will show, the account excludes the possibility that the grounding tree below a truth P could instantiate this structure:

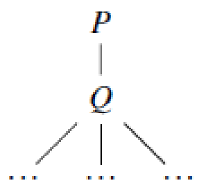

Figure 1

In this structure, there is a single proposition Q which grounds P, such that every other ground of P also grounds Q, and thus grounds P *via* Q, as it were. We might therefore describe it as a *single-conduit* grounding structure. Although it is not obvious that this structure is ever instantiated, there seems to be no independent motivation for taking it to be ruled out by the very *logic* of ground. Moreover, there are perfectly reasonable and attractive views on which there are instances of this structure. The example I shall focus on concerns the existence of singleton sets: one may plausibly take the grounds of the truth that {Socrates} exists to instantiate the single-conduit structure. I conclude that its exclusion of this structure is at least a highly problematic feature of the truthmaker account. So the question arises if there is an attractive way to avoid it.

In the second part of the paper, I show that there is, by developing a modification of the truthmaker account that accommodates the structures in question. The key idea is to recognize an additional *source* of grounding relationships. As I will explain, under Fine's account, grounding relationships turn out to arise in essentially two ways. Given a proposition with a certain set of truthmakers, we can obtain a ground of that truth either by *removing* some of its truthmakers, or by *decomposing* a truthmaker into its proper parts. I propose that we countenance a third way to obtain a ground, in which the truthmakers are not decomposed into proper parts, but replaced by more fundamental states that – as I shall say – *generate* them. We thereby recognize another partial semantic correlate of ground: in addition to the state-proposition relation of truthmaking, a state-state relation of generation. The resulting framework for ground, I argue, is strictly more powerful than the old one. It allows us to straightforwardly capture the previously problematical views, and it allows for every view that could be articulated in the old framework to still be articulated, and in effectively just the same way. Moreover, as I show in a formal appendix, the modified account I propose makes room for single-conduit structures without affecting the more basic parts of the logic of ground; in particular, it yields the same so-called *pure* and *propositional* logics of ground as the old one.[1]

2 THE TRUTHMAKER FRAMEWORK

This section provides the background for the subsequent discussion by describing a formal framework for theorizing about ground, based on the accounts proposed by Fine (2012a; 2012b; 2017b). We assume as given some background language $\mathscr{L}$ in which we can express the propositions whose ground-theoretic relations we are interested in. To study these relations, we use what we may call the *language of ground* $\mathscr{L}_G$ *over* $\mathscr{L}$, consisting of all and only the expressions of the forms

- $\Gamma \leq C$
- $\Gamma < C$
- $A \preccurlyeq C$
- $A \prec C$

where A and C are formulas of $\mathscr{L}$, and Γ is any set of such formulas. The intended interpretation is that $<$ stands for strict full ground, $\leq$ for weak full ground, $\preccurlyeq$ for strict partial ground, and $\preccurlyeq$ for weak partial ground. We shall say more about what these notions are below.

[1] The term 'pure logic of ground' is borrowed from Fine 2012b. Fine's pure logic of ground comprises only relatively simple structural principles of ground, such as transitivity and irreflexivity principles. The propositional logic of ground additionally includes principles connecting ground to the boolean operations of conjunction, disjunction, and negation.

We may then interpret the sentences of $\mathscr{L}$ by assigning propositions to them. In truthmaker semantics, propositions are identified not with sets of worlds, but with sets of states.[2] Like worlds, states are conceived of as specific, or determinate; in particular, whenever a state verifies a disjunction, it does so by verifying at least one of its disjuncts.[3,4] Unlike possible worlds, however, they may be incomplete, leaving open the truth-value of many propositions, and they may be impossible, like the state of a ball being red all over and green all over at the same time.[5]

Instead of the relation of a proposition being true at a world, in truthmaker semantics we appeal to a relation of a proposition being *exactly made true by*, or *exactly verified by*, a state. For a state to (exactly) verify a proposition in this sense, it is required that the state be *wholly relevant* to the truth of the proposition. So the state of snow's being white does not verify the proposition that 2 + 2 = 4, because it is irrelevant to the truth of that proposition. And the state of it being sunny and warm does not verify the proposition that it is sunny or rainy, because it contains as an irrelevant part the state of it being warm, and is therefore not wholly relevant to the truth of the proposition.

As the talk of irrelevant parts implies, states are taken to be ordered by part-whole ($\sqsubseteq$). It is assumed furthermore that given any set of states $T = \{s_1, s_2, \ldots\}$ we may form their fusion $\bigsqcup T = s_1 \sqcup s_2 \sqcup \ldots$, which is taken to be the smallest state containing each of $s_1, s_2, \ldots$ as a part.[6]

A proposition, within truthmaker semantics, is thus a set of states. But we may not wish to count *every* set of states as a proposition. Following Fine, we will require propositions to be non-empty, and to be closed under (non-empty) fusion, so that whenever some states verify a given proposition, so does their fusion.[7] Given an assignment of propositions to the sentences of $\mathscr{L}$, we then need to say which of the sentences of our language of ground $\mathscr{L}_G$ are true. To that end, we first define four relationships of ground over the propositions, corresponding to the four types of grounding statements in $\mathscr{L}_G$ (cf. Fine 2012b, p. 9):

($\leq$) $P_1, P_2, \ldots \leq Q$ iff $s_1 \sqcup s_2 \sqcup \ldots$ verifies Q whenever $s_1, s_2, \ldots$ verify $P_1, P_2, \ldots$, respectively

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[2] Strictly speaking, this is true only of so-called *unilateral* propositions. In many contexts, it is useful to work with a *bilateral* conception of propositions, under which a proposition is identified with a pair of two sets of states, one comprising the states verifying the proposition, the other comprising the states falsifying the proposition. Since the move to bilateral propositions does not affect any of the issues of concern in this paper, for simplicity's sake, it is better to stick to unilateral ones. For details on these issues, see Fine (2017a).

[3] I use the terms 'specific' and 'determinate' interchangeably. On my use, the elimination of disjuncts in a disjunction, and the move from a determinable property to one of its determinates, is correlated with an increase in specificity. The addition of a conjunct to a conjunction, or of differentia to a genus, however, is not. (This contrasts with the usage in (Rosen, 2010: §11), on which all these moves count as increasing specificity, but only the former count as increasing determinacy.)

[4] Strictly speaking, as we shall see below, a state may also verify a disjunction by being the fusion of verifiers of the disjuncts. This subtlety does not affect the point regarding specificity.

[5] Not all applications of truthmaker semantics require impossible states. Indeed, in Fine (2012b) and Fine (2012a), Fine does not envisage inconsistent or even just non-obtaining states. He does do so in Fine (2017b), and I believe that such states are clearly required in order to obtain an adequate account of ground. For the purposes of this paper, not much hangs on this.

[6] The operation of fusion is crucial in particular to the treatment of conjunction: a state is taken to verify a conjunction just in case it is the fusion of a verifier of the one conjunct and a verifier of the other conjunct.

[7] Cf. Fine 2012b, p. 8. The requirement of closure under fusion is the reason why the fusion of verifiers of the disjuncts in a disjunction must also be counted a verifier. In Fine 2017b, pp. 686 and 700ff., a requirement of convexity is also imposed, which demands that whenever a state u lies between two verifiers s and t – that is, $s \sqsubseteq u \sqsubseteq t$ – then u is also a verifier. For present purposes, it does not matter whether convexity is imposed. For some reasons not to impose convexity, see (Krämer and Roski 2015).

($\preccurlyeq$) $P \preccurlyeq Q$ iff $\Gamma, P \leq Q$ for some set of propositions Γ
($<$) $P_1, P_2, \ldots < Q$ iff $P_1, P_2, \ldots \leq Q$ and $Q \preccurlyeq P_i$ for no $i \in \{1, 2, \ldots\}$
($\prec$) $P \prec Q$ iff $P \preccurlyeq Q$ and not $Q \preccurlyeq P$

The grounding statements in $\mathscr{L}_G$ are then considered true iff the corresponding grounding relation holds between the propositions assigned to the relevant sentences of $\mathscr{L}$.

Two comments on these definitions of grounding relations are in order. First of all, it should be noted that the target notions of ground are *non-factive*: there is no requirement that P_1, P_2, …, Q, or the propositions in Γ be true in order for them to instantiate the various grounding relationships.[8] It would be easy enough to define corresponding factive notions, but for present purposes this would needlessly complicate things.

Second of all, it should be noted that the most basic notion of ground defined here, symbolized by $\leq$, besides being non-factive, is somewhat unfamiliar in a further way. In contrast to the intuitive understanding of ground, it is *reflexive*: for any proposition P, we have $P \leq P$. To mark this feature, Fine labels the notion *weak* full ground. In terms of it, and its natural partial counterpart (weak *partial* ground, symbolized by $\preccurlyeq$), he then defines the more intuitive notion of *strict* full ground by imposing a kind of irreversibility requirement, namely that the grounded truth must not partially ground any of the grounding truths. It is the second, strict understanding of ground that we shall mainly focus on below.

Although the precise definition of strict full ground is thus somewhat complicated and indirect, it yields a clear and fairly intuitive picture. In effect, the truthmaker account recognizes two means by which we can move from a proposition to a strict full ground. The first is by (proper) *decomposition* of verifiers. For instance, consider a proposition {u} verified only by the state u. If u may be decomposed into two proper parts s and t, then the corresponding propositions {s}, {t} will jointly strictly fully ground the proposition {u}. The second means is by (strict) *specification*, i.e. eliminating some of the verifiers from a proposition, thereby moving from a proposition for which there are various ways in which it may be true to one which may be true in only some of these. For instance, if u and v are distinct states, then {u} strictly fully grounds $\{u, v, u \sqcup v\}$.[9]

In the case of *singular* grounding, i.e. cases in which a truth is fully grounded by a single truth rather than a plurality of them, no proper decomposition of the grounded proposition's verifiers occurs. So here, specification is the only available means to proceed from a proposition to a strict ground. Correspondingly, for singular grounding the conditions for weak full ground and strict full ground can be simplified:

($\leq_s$) $P \leq Q$ iff $P \subseteq Q$
($<_s$) $P < Q$ iff $P \subset Q$

This completes my exposition of the truthmaker framework for ground as proposed by Fine. I now turn to my objection.

[8] For further discussion of non-factive ground, see e.g. Fine 2012a, pp. 48.
[9] Of course, combinations of the two are possible, as when {s}, {t} jointly strictly fully ground {u, v} under the assumptions in the main text.

3 THE OBJECTION

I will argue that the truthmaker semantics yields too restrictive an account of the possible grounding structures. In particular, as indicated in the introduction, it excludes what I have described in the introduction as single-conduit grounding structures:

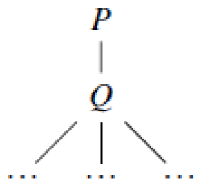

Figure 1

The lines here are to be taken to indicate relationships of strict full grounding, with the grounds lower than what they ground. So in the situation depicted, Q (strictly fully) grounds P, and any other ground of Q grounds P. Making use of the notion of weak full ground, we may summarize the key facts about the structure as follows:

(G.1) $Q < P$
(G.2) For all Γ, if $\Gamma < P$ then $\Gamma \leq Q$

Under the truthmaker account, (G.1) and (G.2) are jointly inconsistent. For assume $Q < P$. Then by ($<_s$), it follows that $Q \subset P$. So P must have a verifier which is not a verifier of Q. Call that verifier x, and let X be the proposition $\{x\}$, verified by x and only by x. Then $X \subset P$, and by ($<_s$) we have $X < P$. But since not $X \subseteq Q$, also not $X \leq Q$. So X is a counter-example to (G.2).

In more informal terms, the point is that any *singular* strict full ground must ground by *specification*, and any truth admitting of proper specification at all can be properly specified in at least two ways. So if a given proposition P has *any* singular strict full ground, it must have at least two:

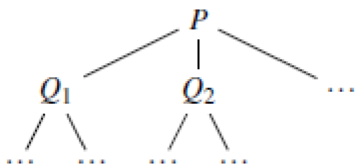

Figure 2

It is important to be clear what this result means. Truthmaker semantics provides an account of the *logic* of ground. If there is no model, within truthmaker semantics, in which a particular kind of grounding structure occurs, then this means that according to truthmaker semantics, the existence of such a structure is ruled out by the logic of ground. So, according to truthmaker semantics, the existence of single-conduit structures is ruled out by the logic of ground. This seems deeply problematic to me.

Before I further explain why, however, there is a subtlety that we should take note of. To characterize the single-conduit grounding structure, we need to *quantify* over all grounds of a given truth, as in (G.2). But within $\mathscr{L}_G$, we cannot formulate such quantifications. As a result, whether or not single-conduit grounding structures are allowed does not manifest itself in whether a particular set of sentences of $\mathscr{L}_G$ is consistent or not. Once we extend $\mathscr{L}_G$ by suitable quantificational devices, however, we can straightforwardly formalize (G.1) and (G.2), and the resulting sentences of our extended language will be jointly inconsistent under the truthmaker approach.

Why should we take this to be problematic? I think there are at least two good reasons. Firstly, there appears to be no *independent* motivation for taking single-conduit structures to be inconsistent. As far as I am aware, none of the principles that have been put forth in the extant literature on the logic of ground provide any reason for ruling out this structure. Indeed, discussion of the logic of ground has so far concerned itself almost exclusively with principles that can be stated within $\mathscr{L}_G$, or a language relevantly like it, which have no prospect of excluding single-conduit grounding. But absent positive reasons for rejecting a given grounding hypothesis as inconsistent with the very logic of ground, it seems the default view should be that the hypothesis is consistent relative to the logic of ground.

Secondly, there is also some direct evidence in favour of the consistency of single-conduit grounding. For there are plausible views which, if correct, provide *concrete instances* of single-conduit grounding. Perhaps the most compelling example concerns the grounds of the existence of a given singleton set, such as {Socrates}. For it seems to be a perfectly reasonable view that

(S.1) Socrates exists < {Socrates} exists
(S.2) For all Γ, if Γ < {Socrates} exists then $\Gamma \leq$ Socrates exists

For (S.1), note that one of the most frequently used examples of a plausible grounding claim is the claim that Socrates' existence grounds the existence of {Socrates} (e.g. Correia & Schnieder 2012, p. 14; Fine 2015, p. 296; Bliss and Trogdon 2016: §4).[10] In most cases, the authors leave unspecified whether they have in mind partial or full ground. But even this would at least seem to indicate that they do not consider its suitability as a paradigm example for a plausible grounding claim to depend on it being interpreted as a claim of partial ground. Moreover, (S.1) has also found explicit endorsement in the literature by Kelly Trogdon (2018, p. 1291).

As for (S.2), I am not aware that the claim has ever been discussed in print, let alone endorsed or rejected. But assuming (S.1), the only way for (S.2) to be false is for there to be a further grounding 'path' towards the existence of {Socrates}, in parallel to that via the existence of Socrates. Perhaps one could come up with potentially reasonable suggestions for such an additional grounding path. Perhaps one might have the view that while the existence of Socrates is sufficient to ground the existence of {Socrates}, there is another full ground of the existence of {Socrates} which includes not just the existence of Socrates, but also some sort of singleton-set-formation principle. But clearly one is not committed to countenancing such an additional grounding path purely because one assents to (S.2).

Aside from the grounds of singleton-existence claims, are there other plausible instances of single-conduit grounding?[11] I think so, though I am sceptical that anyone unconvinced by the example of {Socrates} will be swayed by any of the others. Let me mention just one more

[10] It is clear in each case that the example is not intended as a claim of weak ground.

[11] One may wonder if the same problem does not arise for sets with more members than one. In a sense, it does: just like the truthmaker account rules out that the truth that {Socrates} exists is strictly fully grounded by the truth that Socrates exists, it rules out that the truth that {Socrates, Plato} exists is strictly fully grounded by the truth that Socrates exists and Plato exists. But in this case, there is a related claim of strict full ground which *can* be true under the truthmaker account: that the truth that {Socrates, Plato} exists is *jointly* strictly fully grounded by the two truths that Socrates exists and that Plato exists. As a result, these examples are dialectically less compelling than the singleton case.

pertinent case, which concerns property-ascriptions. One may reasonable hold that true property-ascriptions – truths expressed by instances of ‘a has the property of being F’ – have the corresponding simple predication – expressed by an instance of ‘a is F’ – as their only immediate strict full ground, in the sense that every other ground has to pass through it.[12] If so, they, too, yield instances of single-conduit grounding.

It might be suggested that there are also examples of *logical* single-conduit grounding. For consider the following principles

($L_\vee$) $A < A \vee A$
($L_\wedge$) $A < A \wedge A$
($L_{\neg\neg}$) $A < \neg\neg A$

It is a fairly common view that all instances of these schemata are true. Moreover, in all three cases, it also seems plausible that the ground A should be a sole conduit: all other grounds of $A \vee A$, $A \wedge A$, and $\neg\neg A$ have to pass through A.[13] So the truthmaker account is incompatible with such a view. Indeed, as a number of authors have observed, under the usual truthmaker account, all of A, $A \vee A$, $A \wedge A$, and $\neg\neg A$ have exactly the same verifiers, so ($L_\vee$), ($L_\wedge$), and ($L_{\neg\neg}$) are not merely incompatible with the assumption of A as sole conduit, they are straightforwardly false.[14]

With respect to these examples, however, I think there is a plausible response available to the defender of the truthmaker account. Following Correia (2010), many grounding theorists distinguish between a *worldly* conception of ground on the one hand, and a *representational* conception of ground on the other. It is usually supposed that the former would be considerably less fine-grained than the latter, since it would be sensitive only to differences between truths that concern their relation to worldly items, rather than ones that concern purely representational differences. And cases such as ($L_\vee$), ($L_\wedge$), and ($L_{\neg\neg}$) are commonly used to illustrate the difference between the two conceptions, the thought being that they are plausible only under a representational, but not under a worldly conception of ground (cf. Correia, 2010, pp. 267f), (Fine 2017b, pp. 685f)). Since corresponding truths A and, say, $A \vee A$, plausibly represent the same worldly fact, under a worldly conception, one could not be held to ground the other without allowing us, absurdly, to draw the conclusion that each grounds itself.[15] With this in mind, the truthmaker semantics may then be put forward as adequate only relative to the worldly conception of ground – indeed, this is precisely what is done in Fine (2017b).

In light of this consideration, one might wonder if the same response could be given with respect to the other examples, concerning the existence of singleton sets and property-ascriptions. The idea would be that the truth that Socrates exists and the truth that {Socrates} exists represent the same worldly fact, and similarly the truth that Socrates is wise would be seen as representing the same worldly fact as the truth that Socrates has the property of being wise. However, it seems to me that this response is far less convincing in application to these

[12] A view like this is endorsed by Fine (2012a, p. 67).
[13] This is a straightforward consequence of the so-called *elimination rules* for strict ground proposed in (Fine 2012a, p. 63ff).
[14] ($L_\vee$), ($L_\wedge$), and ($L_{\neg\neg}$), or closely related principles, are endorsed, among others, by Fine (2012a), Rosen (2010), Schnieder (2011) and Correia (2014). Their failure under the truthmaker account is discussed, e.g., by (Krämer 2018, p. 790), who uses it as motivation to develop a different kind of semantics for ground, as well as by Fine himself, e.g. his 2017b, p. 685f and 2012a, p. 74, n. 22.
[15] This is absurd because it would allow us to conclude for *every* truth that it strictly fully grounds itself.

non-logical cases. Firstly, in both cases the plausibility of the claim that the same worldly fact is represented turns on substantive and contentious issues in the philosophy of set theory and the metaphysics of properties, respectively. Other things being equal, we should not expect such issues to be decided by the correct logic of ground. Secondly, any considerations in support of identifying the relevant worldly facts, it seems to me, would have to about the specific subject matter under consideration, i.e. sets and properties. As such, while they may support doubts concerning these specific putative examples of single-conduit grounding, they do not seem to provide much support for the far stronger and more general claim that such grounding structures are excluded already *by virtue of the logical features* of ground. Crucially, the case against this claim developed above does not seem to depend on ground being thought of as representational rather than worldly: no one has ever proposed logical principles for worldly ground that would exclude single-conduit grounding, and the singleton- and property-examples are plausible, though contentious, under both conceptions of ground.

Admittedly, setting aside the question of single-conduit grounding, the truthmaker account of ground has much to be said for it. So it might be suggested that this fact itself gives us reason to accept the truthmaker account, and hence its implication that single-conduit grounding is incompatible with the logic of (worldly) grounding.[16] The plausibility of this suggestion depends, however, on whether this implication is an *essential* consequence of the desirable features of the truthmaker account. So the question is, *must* an alternative semantics that allows for single-conduit grounding be significantly less attractive in other respects, or less in line with a conception of ground as worldly? Or can we give a semantics that allows for single-conduit grounding and which (near enough) matches the truthmaker semantics in the other relevant respects? If we can, then the general virtues of truthmaker semantics do not give much support for the alleged inconsistency of single-conduit grounding. In the next section, I try to develop such an alternative semantics.[17]

4 THE SOLUTION

In a nutshell, the solution I want to propose is to recognize a *third* way in which grounding connections can arise. Like decomposition, it may be described as involving the reduction of a verifier to something more basic. But whereas in the case of decomposition, the reduction of a state s is to a multiplicity of states t_1, t_2, … whose *fusion* is s, in the present case, the reduction of a state s is to a *single* state t. When s may in this way be reduced to the state t, I shall say that t *(strictly) generates* s. For want of another term, I shall call this additional source of grounding connections *(non-mereological) reduction*.

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[16] Thanks to an anonymous referee for pressing me on this.

[17] It is worth noting that the situation seems to me quite different when we consider the putative *logical* instances of single-conduit grounding. There exist two worked-out proposals for a semantics of ground that validates ($L_\vee$), ($L_\wedge$), and ($L_{\neg\neg}$), and renders them instances of single-conduit grounding, due to Correia (2017) and Krämer (2018, 2019). Both are significantly more complicated than the truthmaker semantics, and more importantly, both are significantly less in line with a conception of ground as *worldly*. Correia's semantics is explicitly targeted at the representational conception of ground. Krämer's approach is to see ground as sensitive to whether a worldly fact verifies a given proposition *by* verifying some other proposition, and thus to how a given proposition relates to other propositions. Truthmaker semantics, in contrast, renders ground sensitive only to how a proposition relates to its worldly verifiers. As Krämer himself points out, this constitutes at least one good sense in which his approach and the truthmaker approach are on opposite sides of the worldly/representational divide (cf. his 2019, §4.5).

In more detail, the proposal is as follows. We begin by defining, on the basis of our intuitive notion of (strict full) ground, a binary relation of generation on the states. For any states s and t, we say that s *strictly generates* t (s ⇒ t) iff the proposition verified by s and only s strictly fully grounds the proposition verified by t and only t.[18] We say that s (weakly) generates t (s → t) iff s = t or s strictly generates t. Given some assumptions about grounding, we may then establish parallel principles about generation. In particular, using widely accepted assumptions about grounding, we may establish that weak generation is a partial order, i.e. reflexive, transitive, and anti-symmetric.[19] We then adjust Fine's definition of weak full ground to reflect the idea that generation gives rise to grounding connections. According to the original definition, recall, a proposition P weakly fully grounds a proposition Q iff every verifier of P is *identical* to some verifier of Q. We now replace the appeal to identity by an appeal to (weak) generation. In the general case where we may have a plurality of grounds, the new definition then reads:

(≤*) $P_1, P_2, \ldots \leq Q$ iff $s_1 \sqcup s_2 \sqcup \ldots$ generates some verifier t of Q whenever $s_1, s_2, \ldots$ verify $P_1, P_2, \ldots$, respectively.

We do not make any changes to how the other notions of ground are defined in terms of weak full ground.

The resulting account can easily allow for single-conduit grounding, and thus for the joint truth of (S.1) and (S.2). To see this, let s be the state that Socrates exists, and let t be the state that {Socrates} exists. Let S be the proposition that Socrates exists, and let T be the proposition that {Socrates} exists. We shall assume that

(A.1) $S = \{s\}$
(A.2) $T = \{t\}$

Then in order to obtain (S.1), the claim that S < T, it suffices to make the assumptions that s strictly generates t, and that t is not a part of any state generating s:

(A.3) $s \Rightarrow t$
(A.4) For all states u: not $t \sqcup u \Rightarrow s$

(A.3) straightforwardly ensures that S ≤ T. (A.4) implies that not $T \preccurlyeq S$, which by the definition of strict full ground yields that S < T.[20]

To also obtain (S.2), we need to make sure that any strict full ground of T passes through S. So firstly, we need to ensure that T cannot be grounded by decomposition. We do this by assuming that t is *prime*, in the sense that no fusion of only proper parts of t is identical to t:

[18] Given its intimate connection to grounding, why not just call generation 'grounding'? The reason I prefer to use a different term is that there are two state-level connections – fusion and generation – that bear the same sort of connection to grounding as a relation between propositions. Calling just one of them 'grounding' would suggest an asymmetry in their connection to propositional grounding which is not there. We might wish to describe both of them as relationships of grounding on the level of states, but since their behaviour is rather different in many ways, it seems best to reflect that difference in our terminology.
[19] That is, for all states s, t, u, we have s → s (reflexivity), s → u if s → t and t → u (transitivity), and s = t if s → t and t → s (anti-symmetry). – We shall see in the appendix that some additional assumptions about → have to be made in order that we obtain the right logic of ground, but they enjoy a similar degree of independent plausibility.
[20] *Proof*: Suppose $T \preccurlyeq S$. Then by definition of $\preccurlyeq$, there is a set of propositions Γ such that $\Gamma \cup \{T\} \leq S$. Then let X result from picking one verifier from each member of Γ. By definition ≤, it follows that $\bigsqcup(X \cup \{t\}) \rightarrow s$. But $\bigsqcup(X \cup \{t\}) = \bigsqcup X \sqcup t$, so it follows that $\bigsqcup X \sqcup t \rightarrow s$, contrary to (A.4).

(A.5) For all sets of states T: if $t = \bigsqcup T$ then $t \in T$

Finally, we need to make sure that all grounding of T by reduction goes via S. To that end, we assume that every state generating t does so via s:

(A.6) For all states u: if $u \Rightarrow t$ then $u \rightarrow s$

Given the assumptions (A.1)–(A.6), we can derive the truth of both (S.1) and (S.2), and thus obtain that the grounding tree for the existence of {Socrates} instantiates the single-conduit structure.[21]

It should be noted, moreover, that assumption (A.1) is not essential to getting (S.1) and (S.2) to come out true. One might hold instead that there are many verifiers of the proposition that Socrates exists, corresponding perhaps to the many ways for there to be some simples arranged Socrates-wise. One then only needs to adjust (A.3), (A.4), and (A.6) accordingly, making sure these states are exactly the states that strictly generate t, and that t does not help generate any of them.[22]

It is also worth pointing out that the primeness assumption (A.5) is compatible with t having proper parts, and in particular with s being a proper part of t. It only rules out that s is a supplemented proper part of t, i.e. that there is a further proper part u of t such that $s \sqcup u = t$. But the standard assumptions in truthmaker semantics about the mereology of states allow for these kinds of unsupplemented proper parts.[23]

So the modification I propose does some good: it makes room for more plausible grounding structures than the original truthmaker semantics. It remains to show that it does not do more harm than good. Fortunately, it turns out that many, if not all, of the desirable features of the original account are preserved under the modification. The change in the logic of ground it brings about is rather *local*: allowing for single-conduit grounding does not force us to give up any of the logical principles for ground that are typically considered in the debate. In particular, as I show in the appendix, we can retain exactly the same pure and propositional logic of ground that the original truthmaker account yields. Moreover, the modified semantics seems to fit just as well as the original one with a conception fo ground as a *worldly* relation. For just like the original account, it renders ground sensitive only to how ground and groundee relate to their worldly verifiers, and how these relate to one another. The only change is that we are taking into account an additional relation of generation between the worldly verifiers themselves.

[21] *Proof*: Suppose $\Gamma < T$. Then $\Gamma \leq T$, and not $T \preccurlyeq Y$ for any $Y \in \Gamma$. We wish to show that $\Gamma \leq S$. So let $x = \bigsqcup X$ for any X obtained by picking one verifier from each member of Γ. Then what we need to show is simply that $x \rightarrow s$. Since $\Gamma \leq T$, $x \rightarrow t$. Now suppose for contradiction that $x = t$. Then $t = \bigsqcup X$, so by (A.5), $t \in X$. Hence $t \in Y$ for some $Y \in \Gamma$. But then $T \leq Y$, and hence $T \preccurlyeq Y$, contrary to our assumption. So since $x \rightarrow t$ and $x \neq t$, it follows that $x \Rightarrow t$. But then by (A.6), $x \rightarrow s$, as desired.

[22] If we assume the plausible seeming principle that no state generates any of its proper parts, we may actually derive (A.6) or its adjusted counterpart from the other assumptions. Thus, assume for contradiction that $t \sqcup u \rightarrow s$. By transitivity of $\rightarrow$ and (A.3), $t \sqcup u \rightarrow t$. Then by the principle just described, t cannot a proper part of $t \sqcup u$ and thus must be identical to $t \sqcup u$. But then $t \rightarrow s$, and hence by anti-symmetry of $\rightarrow$, $s = t$, in contradiction to (A.3). – It would be very interesting to examine what further principles might govern the interaction of $\rightarrow$ and $\sqsubseteq$, but this is something I shall have to leave for future work, although some related issues briefly come up in the formal appendix.

[23] Indeed, it is a natural hypothesis that s is a proper part of t, and perhaps even an unsupplemented proper part of t, whenever $s \Rightarrow t$. But as mentioned in the previous footnote, these issues call for a more extended discussion, which will have to wait for another occasion.

The modification does, of course, introduce some additional complexity into the theory. Instead of dealing just with the part-whole relation on the states, we now have to also deal with a second relation of generation, and the way the two relations interact. But as the appendix shows, the assumptions needed here are few, and they are fairly simple and straightforward counterparts to plausible assumptions about ground.[24] So I think the advantages that the modification offers are well worth the additional complexity. Moreover, the general picture of how grounding relations arise that the account offers is still a very simple and intuitive one: starting from derivative truths, we work towards more fundamental ones by *eliminating* a verifier, thereby obtaining a *more specific* description of reality, or by *reducing* a verifier, either to a generating state, or to a collection of states of which it is the fusion, thereby obtaining a description of *more basic elements* of reality.

It should be noted that there is a sense in which part of my proposal is a *generalization* of the truthmaker semantics, rather than a competitor to it. For we may distinguish between two components of the proposal. One is an adjusted definition of a truthmaker interpretation, or *model*, of the language of ground, and a definition of what it is for a sentence of that language to be true in a model:[25] in contrast to the original truthmaker models, ours include a relation of generation in addition to the part-whole relation on the states and the truth-conditions for grounding claims appeal to that additional relation. Now we may understand the notion of logical consequence as truth-preservation across all *admissible* models. There is then a separate question which of our models are admissible. In particular, there is the question whether any models that give rise to single-conduit structures are admissible. So the second component of the view I have proposed is a characterization of the admissible models under which we do obtain examples of single-conduit structures. But we can of course retain the first component of my proposal and consider other possible characterizations of the class of admissible models. For instance, if we consider admissible only models in which generation coincides with identity, we obtain exactly the same account of the logic of ground that the original truthmaker semantics gives rise to.

There is an instructive parallel to this situation in the theory of metaphysical modality. In the simplest form of possible world semantics for metaphysical modality, we do not appeal to any kind of accessibility relation on the worlds: necessity is simply understood as truth in all possible worlds. By introducing an accessibility relation, and taking necessity to be a matter of truth in all accessible worlds, we obtain a more general framework. It collapses into the old one under the assumption that every world is accessible from every other world. But it can also capture other possible views, under which the assumptions about accessibility are weaker. So without a claim concerning which models are admissible, we may see my proposal as standing to standard truthmaker semantics in the same way that possible world semantics with an accessibility relation stands to the simpler version without accessibility.

[24] It is perhaps worth nothing that one could also avoid working with two relations on states, and instead postulate just one relation of grounding between sets of states and states. The idea would be that the generation relation could be recovered as the special case of a singleton set grounding a state, and the fusion operation could be recovered by letting the fusion of a set of states be the least state, with respect to grounding, which is grounded by the given set of states. From the fusion operation, we could then recover parthood by letting $s \sqsubseteq t$ iff $s \sqcup t = t$. I am skeptical, however, that any real simplification can be achieved in this way. As far as I can see, the assumptions required about this kind of grounding relation on the states are considerably less simple and less natural than the ones needed for parthood and generation.

[25] The precise definitions are given in the appendix.

Finally, it might be objected that my modification does not preserve the *reductive* nature of the original account, since the novel element of the generation relation is explicitly defined in terms of a prior, intuitive concept of ground, which is taken as primitive. It is true that my proposed account cannot satisfy reductive ambitions. Along with most participants in the contemporary debate, I take the concept of ground to resist reductive analysis. But I maintain that even the original truthmaker semantics should not be regarded as offering any kind of conceptual reduction of ground (nor is there any indication that Fine meant to propose it in such a spirit). The most compelling reason for this is that the notion of exact truthmaking is so closely related to grounding that it seems implausible that our grasp of this notion is independent of our grasp of the notion of ground. Indeed, Fine himself suggests that truthmaking may be explained in terms of ground, rather than the other way round:

> [I]ndeed, we might think of the notion of exact verification as being obtained through a process of ontological and semantic ascent from a claim of ground [to the effect that A_1, A_2, … ground C]. For we first convert the statements A_1, A_2, … into the corresponding facts f_1, f_2, … (that A_1, A_2, … obtain) and then take the sum f of the facts f_1, f_2, … to be an exact verifier for the truth of C. (Fine 2017c, §3)[26]

But if the semantics cannot serve reductive ambitions, exactly what is it good for?[27] There are a number of important uses for a formal semantics which do not depend on its reductive potential. Some of them were already in evidence in this paper, others we have hinted at. A formal semantics provides the resources to define relations of *consequence* – truth-preservation across all (admissible) models – and *consistency* – truth in at least one (admissible) model. It thereby allows us to evaluate proposed deductive systems for the relevant languages for soundness and completeness. It may also allow us to connect the question of the consistency of a particular set of grounding claims with specific conditions on models. We can then see what impact it has on the logic if we allow or disallow such models, and thereby obtain further evidence for or against the consistency of the grounding claims in question. Thus, in this paper, we have connected the question of the consistency of single-conduit grounding structures, or sets of sentences characterizing them, to specific conditions on the behaviour of the generation and parthood relations on the states, and we have examined the impact on the logic of ground of allowing for models of the relevant kind.[28,29]

[26] The idea that Fine sketches here is developed in more detail in Pleitz (ms).

[27] Thanks to an anonymous referee for urging me to elaborate on this. The question is of course as pressing for Fine as it is for me, given that the original truthmaker semantics also cannot satisfy reductive ambitions. Fine briefly addresses the point in his 2012b, p. 2, offering a similar account of the benefits of a formal semantics to the one I give.

[28] There are other examples in the literature in which the truthmaker semantics is put to this kind of role. For instance, Leuenberger (2019) discusses whether a truth can have a strict partial ground without having any strict full ground. He notes that within truthmaker semantics, this turns on whether we allow for states that have proper parts, but are prime in the sense defined above.

[29] Again, the modal analogy may be illuminating. By varying the constraints on the accessibility, a range of different candidate modal logics are obtained, and a number of interesting modal hypothesis are in this way connected to different such logics. For instance, Salmon (1989) has defended the view that some claims may be impossible, but possibly possible. This is incompatible with the modal logic S5, which is validated by the simple possible world semantics without accessibility. Within the semantics with an accessibility relation, the consistency of Salmon's examples turns on whether accessibility is allowed to be non-transitive, and we can study exactly what sort of modal logics we can obtain if we make this assumption.

6 CONCLUSION

My aim in this paper was to develop and defend a modification of the truthmaker semantics for ground as developed by Kit Fine. I first argued that Fine's account yields a problematically restrictive view of the possible grounding structures, by ruling out single-conduit structures: structures with unique, singular, immediate strict full grounds. The account thereby excludes as inconsistent with the very logic of ground an otherwise natural and attractive view of how the existence of a singleton set like {Socrates} is grounded in the existence of its member. I then described a modified version of the semantics which avoids this difficulty by countenancing an additional source of grounding relationships in the form of a relation of *generation* between truthmakers. At the same time, as I argued in the previous section, the modification preserves most of the desirable features of the original account. I conclude that the evidence available so far favours the view of single-conduit grounding as consistent, and hence my modification over the original truthmaker account. Moreover, even if we leave open this question of consistency, the proposed modification constitutes progress, since it provides a strictly more general formal framework within which we may articulate, study and compare competing accounts both of the general logic of ground, and of more specific questions concerning what grounds what.[30]

APPENDIX

In this appendix, I show that my proposed modification of the truthmaker semantics for ground does not lead to a change in either the pure or the propositional logic of ground. I do this by showing that for every interpretation of the language of ground under the old semantics there is an equivalent interpretation of the language under the new semantics, and vice versa.

First, we give the definition of a state-space, the basic structure in the original truthmaker semantics.[31]

Definition 1. A *state-space* is any pair $(S, \sqsubseteq)$ such that

1. S is a non-empty set
2. $\sqsubseteq$ is a partial order on S such that every subset of S has a least upper bound[32] with respect to $\sqsubseteq$ in S

The least upper bound of any set $T = \{t_1, t_2, \ldots\}$ states is their fusion $\bigsqcup T = t_1 \sqcup t_2 \sqcup \ldots$ Following Fine, we assume that even the empty set has a fusion, which we call the nullstate and denote by $\square$. It is easily verified that the nullstate is part of every state, and that $s \sqcup \square = s$ for all states s.

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[31] For the formal details of Fine's truthmaker semantics, see the appendices of Fine (2017a,b). The latter paper also briefly discusses the application to ground. My presentation of the Finean account is based on these most recent papers rather than Fine (2012b). The differences between the different versions of Fine's account are inessential for our purposes.

[32] A state $s \in S$ is an *upper bound* (w.r.t. $\sqsubseteq$) of $T \subseteq S$ iff $t \sqsubseteq s$ for all $t \in T$, and it is a *least* upper bound of T iff $s \sqsubseteq u$ for every upper bound u of T. It is routine to show that least upper bounds are unique if they exist.

The basic structure of the modified semantics is what I call a *generation-space*:

Definition 2. A *generation-space* is any triple (S, $\sqsubseteq$, $\rightarrow$) such that

1. (S, $\sqsubseteq$) is a state-space
2. $\rightarrow$ is a partial order on S such that
   a. if $s_1 \rightarrow t_1$ and $s_2 \rightarrow t_2$ and … and $t_1 \sqcup t_2 \sqcup \ldots \rightarrow v$ then $s_1 \sqcup s_2 \sqcup \ldots \rightarrow v$
   b. if $s \rightarrow t \sqcup u$ then $s = s_1 \sqcup s_2$ for some s_1, s_2 with $s_1 \rightarrow t$ and $s_2 \rightarrow u$

Note the two new conditions (2.a) and (2.b) constraining the interaction between $\rightarrow$ and $\sqsubseteq$. (2.a) is a *cut* constraint, asserting a strong form of transitivity for generation, parallel to the cut rule in Fine's pure logic of ground, which says that given $\Gamma_1 \leq A_1$, $\Gamma_2 \leq A_2$, …, and $A_1, A_2, \ldots \leq C$, we may infer that $\Gamma_1, \Gamma_2, \ldots \leq C$ (Fine 2012b, p. 5). The second constraint (2.b) says that a state only gets to generate a fusion of two states by being a fusion of generators of the states being fused. Its counterpart on the level of propositions is the principle that a proposition only gets to weakly fully ground a conjunction by being the conjunction of weak full grounds of the conjuncts, a valid rule of the propositional logic of ground under the truthmaker account.

We note some useful facts about generation-spaces.

Proposition 1. If (S, $\sqsubseteq$) is a state-space, (S, $\sqsubseteq$, =) is a generation-space.

Proof: It suffices to show that the identity relation satisfies the conditions on $\rightarrow$ in a generation-space. Identity is obviously reflexive, transitive, and anti-symmetric, and hence a partial order. Moreover, if $s_1 = t_1$ and $s_2 = t_2$ and . . . and $t_1 \sqcup t_2 \sqcup \ldots = v$ then clearly $s_1 \sqcup s_2 \sqcup \ldots = v$. Finally, suppose $s = t \sqcup u$. Then let $t = s_1$ and $u = s_2$ to show that there are states s_1, s_2 with $s_1 \rightarrow t$ and $s_2 \rightarrow u$.

Proposition 2. Let (S, $\sqsubseteq$, $\rightarrow$) be any generation-space, and let $s_1, t_1, s_2, t_2, \ldots$ be members of S. Then if $s_1 \rightarrow t_1$ and $s_2 \rightarrow t_2$ and …, also $s_1 \sqcup s_2 \sqcup \ldots \rightarrow t_1 \sqcup t_2 \sqcup \ldots$

Proof: From Reflexivity of $\rightarrow$ by an application of the constraint (2.a), setting $v = t_1 \sqcup t_2 \sqcup \ldots$

Definition 3. Let (S, $\sqsubseteq$, $\rightarrow$) be any generation-space. For T $\subseteq$ S, let the *closure under generation* T^G of T be the set $\{s \in S: s \rightarrow t \text{ for some } t \in T\}$.

Proposition 3. Let (S, $\sqsubseteq$, $\rightarrow$) be any generation-space, and let T be a non-empty subset of S which is closed under non-empty fusion. Then T^G is also non-empty and closed under non-empty fusion.

Proof: Non-emptiness follows from the non-emptiness of T and the reflexivity of $\rightarrow$. For closure, suppose U is a non-empty subset of T^G. Then for each $u \in U$, there is a state $s_u \in T$ with $u \rightarrow s_u$. By closure of T, the fusion of all s_u is also in T. By proposition 2, the fusion of all $u \in U$ generates the fusion of all the s_u, and hence is also in T^G.

Now given any language $\mathscr{L}$, let the *language of ground* $\mathscr{L}_G$ *over* $\mathscr{L}$ consist of all and only the expressions of the forms

- $\Gamma \leq C$
- $\Gamma < C$
- $A \preccurlyeq C$
- $A \prec C$

where A and C are formulas of $\mathscr{L}$., and Γ is any set of such formulas.

Consider first the so-called pure logic of ground, which studies only the structural features of ground, without attention to the internal makeup of the relata of ground. Let $\mathscr{L}^P$ be a non-empty set of atomic sentences. Given a state-space $(S, \sqsubseteq)$, say that $\mathscr{M}_O = (S, \sqsubseteq, I)$ is an *old model* of $\mathscr{L}^P{}_G$ iff I maps every member of $\mathscr{L}^P$ to a non-empty subset of S which is closed under (non-empty) fusion. Likewise, given a generation-space $(S, \sqsubseteq, \rightarrow)$, say that $\mathscr{M}_N = (S, \sqsubseteq, \rightarrow, I)$ is a *new model* of $\mathscr{L}^P{}_G$ iff I maps every member of $\mathscr{L}^P$ to a non-empty subset of S which is closed under fusion. We give the obvious truth-conditions for the four types of grounding statements in $\mathscr{L}^P{}_G$, using our adjusted clause for weak full ground for truth in a new model. For Φ a subset and φ a member of $\mathscr{L}^P{}_G$, say that Φ entails$_O$ (entails$_N$) φ iff φ is true in every old (new) model in which every member of Φ is true.

Lemma 4. For every old model there is a new model in which exactly the same sentences of $\mathscr{L}^P{}_G$ are true.

Proof: Let $\mathscr{M}_O = (S, \sqsubseteq, I)$ be an old model and let $\varphi \in \mathscr{L}^P{}_G$. Let $\mathscr{M}_N = (S, \sqsubseteq, =, I)$, By proposition 1, $\mathscr{M}_N$ is a new model. Since all forms of partial and strict ground are defined in the same way in terms of weak full ground for both old and new models, it suffices to consider the case in which φ is of the form $\Gamma \leq C$. Then if φ is true in $\mathscr{M}_O$, any fusion of verifiers of the members of Γ is a verifier of C. Since the generation relation of the new model is reflexive, it follows that any fusion of verifiers of the members of Γ generates a verifier of C, and hence that φ is true in $\mathscr{M}_N$. But if φ is not true in $\mathscr{M}_O$, then some fusion t of verifiers of the members of Γ is not a verifier of C. Since the generation relation of the new model is the identity relation, it follows that t does not generate a verifier of C, and hence that φ is not true in $\mathscr{M}_N$.

Lemma 5. For every new model there is an old model in which exactly the same sentences of $\mathscr{L}^P{}_G$ are true.

Proof: Let $\mathscr{M}_N = (S, \sqsubseteq, \rightarrow, I)$ be a new model and let $\varphi \in \mathscr{L}^P{}_G$. Let J map every member A of $\mathscr{L}^P$ to $I(A)^G$, the closure under generation of I(A). From proposition 3, it follows that $\mathscr{M}_O = (S, \sqsubseteq, I)$ is an old model. Again, since all forms of partial and strict ground are defined in the same way in terms of weak full ground for both old and new models, it suffices to consider the case in which φ is of the form $\Gamma \leq C$. For explicitness, let us write φ as $A_1, A_2, \ldots \leq C$. If $s \in I(A)$, we say that s is a verifier$_I$ of A, and likewise for $_J$.

Suppose first that φ is true in $\mathscr{M}_N$, and let $s_1, s_2, \ldots$ verify$_J$ $A_1, A_2, \ldots$, respectively. We need to show that $s_1 \sqcup s_2 \sqcup \ldots$ verifies$_J$ C. By definition of J, each s_i generates a verifier$_I$ t_i of the corresponding A_i. Since φ is true in $\mathscr{M}_N$, the fusion of the t_i generates a verifier$_I$ of C. By the cut constraint (2.a), it follows that the fusion of the s_i generates a verifier$_I$ of C. By the definition of J, we can thus infer that the fusion of the s_i is a verifier$_J$ of C, and hence that φ is true in $\mathscr{M}_O$, as desired.

Suppose now that φ is not true in $\mathscr{M}_N$. Then let $s_1, s_2, \ldots$ be verifiers$_I$ of $A_1, A_2, \ldots$, respectively, such that the fusion of the s_i does not generate a verifier$_I$ of C. By definition of J, the fusion of the s_i is not a verifier$_J$ of C, while each s_i still verifies$_J$ the corresponding A_i. It follows that φ is not true in $\mathscr{M}_O$, as desired.

Theorem 6. For Φ any subset and φ any member of $\mathscr{L}^P{}_{G,}$ Φ entails$_O$ φ iff Φ entails$_N$ φ.

Proof: From the previous two lemmas.

We now turn to the *impure*, propositional logic of ground. To avoid irrelevant distractions, we set aside negation, and focus purely on conjunction and disjunction.[33] First, we define the operations of conjunction and disjunction on propositions.

Definition 4. Let $(S, \sqsubseteq)$ be any state-space, and let P and Q be non-empty subsets of S closed under non-empty fusion. Then

- $P \wedge Q = \{s \sqcup t: s \in P \text{ and } t \in Q\}$
- $P \vee Q = (P \cup Q) \cup (P \wedge Q)$

Now let the language $\mathscr{L}^I$ be the closure of $\mathscr{L}^P$ under the connectives $\wedge$ and $\vee$. Old and new models of $\mathscr{L}^I_G$ are just like their counterparts of $\mathscr{L}^P_G$, but with I extended to the complex formulas in $\mathscr{L}^I$ in the obvious way, letting $I(A \wedge B) = I(A) \wedge I(B)$ and $I(A \vee B) = I(A) \vee I(B)$.

As before, we define 'old' and 'new' entailment relations in terms of the corresponding classes of models. To extend theorem 6 to $\mathscr{L}^I_G$, it suffices to show that closure under generation distributes over conjunction and disjunction. More precisely:

Lemma 7. Let $(S, \sqsubseteq, \rightarrow)$ be any generation-space, and let P and Q be non-empty subsets of S closed under non-empty fusion. Then

1. $(P \wedge Q)^G = P^G \wedge Q^G$
2. $(P \vee Q)^G = P^G \vee Q^G$

Proof: For 1., suppose $s \in (P \wedge Q)^G$, so $s \rightarrow t \sqcup u$ for some $t, u \in P, Q$. By constraint (2.b) on $\rightarrow$, there are states s_t, s_u with $s = s_t \sqcup s_u$ and $s_t \rightarrow t$ and $s_u \rightarrow u$. Then $s_t \in P^G$ and $s_u \in Q^G$, hence $s \in P^G \sqcup Q^G$. Conversely, suppose $s \in P^G \wedge Q^G$, so $s = t \sqcup u$ for some t, u generating verifiers of P, Q, respectively. By proposition 2, s generates the fusion x of these verifiers of P and Q. By the definition of conjunction, x verifies $P \wedge Q$, hence $s \in (P \wedge Q)^G$.

For 2., suppose $s \in (P \vee Q)^G$, so s generates a verifier of P, or a verifier of Q, or a verifier of $P \wedge Q$. In the first case, $s \in P^G$. In the second case, $s \in Q^G$. In the third case, by the reasoning before, $s \in P^G \wedge Q^G$. So by the definition of disjunction, in all three cases, $s \in P^G \vee Q^G$. Conversely, suppose $s \in P^G \vee Q^G$. Then either $s \in P^G$, in which case s generates a verifier of P, and hence of $P \vee Q$, or $s \in Q^G$, in which case s generates a verifier of Q, and hence again of $P \vee Q$, or $s \in (P^G \wedge Q^G)$, in which case by the above reasoning, s generates verifier of $P \wedge Q$, and hence again of $P \vee Q$. So in all three cases, $s \in (P \vee Q)^G$.

Theorem 8. For Φ any subset and φ any member of $\mathscr{L}^I_G$, Φ entails$_O$ φ iff Φ entails$_N$ φ.

Proof: From the obvious counterparts to lemmas 4 and 5. The proof of the first, that to every old model there is an equivalent new one, carries over without any changes. For the proof of the second, we construct the old model from the new one in the same way as before, letting J assign to each atomic members A of $\mathscr{L}^I$ the closure under generation of its interpretation I(A) in the new model. We then conclude from lemma 7 that for *every* formula A of $\mathscr{L}^I$, the interpretation J(A) in the old model equals $I(A)^G$. The result then follows by exactly the same reasoning as before.

[33] The treatment of negation introduces technical complexities which are irrelevant to our present concerns. For some ways to deal with negation within truthmaker semantics, see (Fine 2017a, pp. 629ff), (Fine 2017a, pp. 634f, 658), and (Fine 2014, pp. 554ff).

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