THE WHOLE TRUTH

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Abstract. We often care not just whether an account of some subject matter is correct, but also whether it is complete—the whole truth, as we might say. The paper criticizes extant intensional explications of the notion of a whole truth by showing that they yield implausible results in an important range of cases. The difficulty is traced to the inability of an intensional framework to adequately capture constraints of relevance imposed by an intuitive understanding of the whole truth. I go on to develop and defend a novel account of what it is for a truth P to be the whole truth with respect to a subject matter: roughly speaking, it is for every fact pertaining to the subject matter to be relevant to making P true, or equivalently, for P to relevantly entail every truth pertaining to the subject matter. The proposal is formally spelled out within the framework of truthmaker semantics as developed by Kit Fine in a series of recent publications. As part of this, a novel, truthmaker-based semantics for the totality operator '. . . and that's it' is sketched and argued to be superior to previous intensional accounts.

1. Introduction

In both scientific and everyday contexts, we often care not just whether a given account of some subject matter is *correct*—whether it describes its subject matter accurately—but also whether it is *complete*—whether it describes its subject matter exhaustively. In this paper, I propose a novel account of what it is for a proposition to be both correct and complete—*the whole truth*, as one might say—with respect to a given subject matter. The most distinctive feature of the account is that it imposes requirements of *relevance*. Roughly speaking, on the view to be developed, for P to be a whole truth with respect

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to a given subject matter is for every fact pertaining to that subject matter to be *relevant* to making P true, or equivalently, for P to *relevantly* entail every truth pertaining to the subject matter.

Section 2 clarifies, by means of examples, the notion of a whole truth that I aim to capture, and uses the examples to motivate two preliminary informal characterizations of that notion. I then turn to the task of making these characterizations formally precise. Section 3 presents a natural formalization within the intensional framework of possible worlds semantics, and argues that it yields implausible results in an important range of cases. The difficulty is traced to the inability of an intensional framework to adequately capture the relevance constraints imposed by an intuitive understanding of the whole truth. This diagnosis is then used to rule out as unpromising a number of otherwise natural seeming strategies to refine the possible worlds analysis. Section 4 introduces the hyperintensional framework of truthmaker semantics, as recently developed by Kit Fine (esp. Fine (2017a,b)) and uses it to develop alternative formalizations of our informal characterizations of the whole truth. It is then argued in section 5 that this proposal avoids the difficulties faced by the previous accounts. Section 6 shows how to extend the truthmaker-based account by so-called totality operators to capture an important weakening of the previous notion of a whole truth. The operators are also given a hyperintensional, relevantist treatment, which is again argued to be superior to extant intensional accounts. Section 7 concludes.

2. Some examples, and two informal characterizations

The notion of a complete truth has application in a wide range of contexts. A first, very large class of examples concerns the interpretation of answers to wh-questions. Thus, consider the following question-answer pair:

Sara:: What did you have for breakfast?

Jack:: I had eggs, bacon, and orange juice.

At least in typical contexts, the natural interpretation of Jack's answer to Sara's question is as *exhaustive*. So interpreted, Jack's answer is incompatible with his also having had coffee for breakfast. Correspondingly, Jack's answer is wholly appropriate only if it is a *complete* truth with respect to the subject matter of what Jack had for breakfast. Thus, the evaluation of Jack's answer as appropriate or otherwise involves at least a tacit application of a concept of a complete truth. Parallel comments apply for most answers to wh-questions.[1]

[1] This observation and the concept of a complete, or *exhaustive* answer to a question constitute an important topic in the theory of questions within linguistics. The classical reference is Groenendijk

There are also a number of more specialized contexts in which the notion of a whole truth is of particular significance. Perhaps the most notorious of these is the context of legal testimony. It often matters a great deal not just whether a witness's account is true, but also whether it is in some relevant sense exhaustive—as reflected in the oath to tell the truth, *the whole truth*, and nothing but the truth.[2] In addition, there are several more *theoretical* contexts in which the notion of a complete truth has application. For instance, in metaphysics, a number of important hypotheses may be construed as claims concerning what *kind* of truth might be complete. Thus, physicalism may be understood, in a first approximation, as the claim that a complete physical description of the world is by itself already a complete description of the world *full stop*.[3] A second possible application in metaphysics concerns the grounds of universal quantification. It has been argued that any true universal quantification $\forall x\, Fx$ is fully grounded by the totality of its instances Fa, Fb, ... together with the totality claim that a, b, ... are all the things that exist.[4] That latter claim might plausibly be understood as the claim that the conjunction of the proposition that a exists, and the proposition that b exists, and ... is a complete truth with respect to the question of what individuals exist.

Another class of examples, from logic and epistemology, concerns defeasible forms of reasoning. Since birds can normally fly, from the premise that Tweety is a bird one may defeasibly infer that Tweety can fly. In making this inference, we might say, one tacitly conjectures that the premise constitutes a *complete truth* with respect to those of Tweety's properties that are relevant to Tweety's being able to fly—and so in particular, that Tweety is not a penguin, and does not have broken wings, etc.[5] A related issue

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and Stokhof (1984). It should be pointed out that for some questions, like 'Where can I buy an international newspaper?', incomplete answers—often called 'mention-some' answers in contrast to complete, 'mention-all' answers—do seem entirely felicitous. It is controversial whether this is due purely to pragmatic factors, or whether there are semantic differences involved. That question is immaterial for our purposes though.

2 Exactly how the phrase 'the whole truth' is to be understood in this context is admittedly a delicate matter. Since the witness will typically not have knowledge of every aspect of the situation under discussion, the requirement to tell the whole truth is presumably not that of giving an accurate and complete account of the situation. Rather, the requirement might be to accurately and completely report one's pertinent *knowledge* about the situation. By doing so, one then implicitly specifies a whole truth with respect to the matter of what pertinent knowledge one possesses.

3 This has been stressed by (Leuenberger, 2014: p. 529).

4 For a statement of the view in the context of the contemporary theory of ground, see (Fine, 2012a: pp. 60ff).

5 In the theory of defeasible reasoning, this assumption is often called the closed-world assumption. Similar forms of completeness-assumptions may be seen to motivate John McCarthy's influential system for defeasible reasoning, the logic of circumscription (McCarthy (1980, 1986)).

shows up in the context of Bayesian epistemology. Assume that Bob's credence in the proposition that Tweety can fly given that Tweety is a bird is high—say, 0.9. Then according to standard Bayesianism, when Bob learns that Tweety is a bird, he should update by conditionalizing on this newly obtained evidence and set his credence in the proposition that Tweety can fly to 0.9. However, this is so only under the assumption that Tweety's being a bird is *all* that Bob learns in the situation under discussion—and so in particular that Bob does not also learn that Tweety is a penguin (and that penguins can't fly).[6] Whether Bob's setting his credence to 0.9 is rational thus depends on whether the truth that Bob learned that Tweety is a bird is a complete truth with respect to the question of what Bob learned in the relevant situation.[7]

What, then, does it mean for a truth to be complete with respect to a subject matter? There are two natural and complementary strategies we may pursue in trying to clarify this. The first strategy is to try to say how the truth must relate to *other truths* in order to be complete. I shall call this the *horizontal* strategy. The obvious first idea is that it must *entail every other truth* that pertains to the subject matter in question.[8] For suppose there is a truth about the relevant subject matter that is not entailed by a candidate whole truth P. Then P is compatible with a false hypothesis pertaining to that subject matter, and this would seem to constitute a way in which P falls short of completeness. If, on the other hand, P does entail every truth pertaining to the subject matter, then it is prima facie plausible to conclude that P is complete with respect to that subject matter.

The second strategy is to try to say how the truth must relate to *the world* in order to be complete. I shall call this the *vertical* strategy. Here a natural suggestion is that a candidate whole truth P must *report every fact*—construed as a worldly item—that

6 In Bayesian epistemology, this is captured in the requirement of total evidence, to the effect that one must always update on one's total evidence.

7 Another area in which the notion of a complete truth with respect to some subject matter plays a prominent role is of course that of mathematical logic, when we ask whether a given formal theory constitutes a complete description of the mathematical structures that it aims to capture. I do think that at least some of the notions of completeness in play here belong to the same family as the notions considered here, and that there is considerable interest in applying the account I will develop to the mathematical cases. Nevertheless, I shall set these cases aside for the purposes of this paper, both because they introduce too many additional complications and because for this area, there are perfectly clear and precise formal explications of completeness available, so it is less clear that there is any need for a novel account.

8 Note that 'pertaining' should here be understood as 'wholly pertaining'. If a given truth P pertains in part to the relevant subject matter m and in part to some other subject matter m', then a complete truth about m may unproblematically fail to entail P simply because it is quiet on m'.

pertains to the relevant subject matter.[9] Thus, whenever it pertains to the subject matter that a certain fact obtains, then *P* must state that that fact obtains. If it does not, then this would seem to constitute a way in which *P* falls short of completeness. If, on the other hand, *P* does report every fact pertaining to the relevant subject matter, then it seems prima facie plausible to conclude that *P* is complete with respect to that subject matter.

It might be objected, certainly with respect to the first, and possibly with respect to the second suggestion, that it is too demanding. For note that one of the truths with respect to Jack's breakfast, for example, is (we may assume) that Jack did not eat a crocodile for breakfast. So by our first suggestion, a complete truth with respect to the subject matter of what Jack had for breakfast would have to entail this truth. But at least on one natural understanding of completeness, this is not so. On that understanding, it is sufficient, roughly speaking, if a truth entails what Jack *did* eat for breakfast, and it need not also entail what Jack did *not* eat.[10] Similarly, it might be held that among the facts pertaining to the subject matter of what Jack had for breakfast is the fact that he did not eat a crocodile.[11] But intuitively, in one good sense of 'complete', a complete truth with respect to that subject matter need not report that fact: it need only report the facts concerning what Jack did eat, and may omit mention of facts concerning what he didn't eat.

The issue arises also for some of the theoretical applications of the notion of a complete truth mentioned above. The complete physical description of the world does not entail the truth that there are no demons (construed as non-physical objects). But that does not, it seems, prevent it from being complete in the, or at least a, metaphysically important sense of 'complete'. Again, this sense seems only to require a complete truth

[9] For reasons analogous to those given in the previous footnote, 'pertaining' should be read as 'wholly pertaining'. I shall henceforth take this reading for granted.—My talk of a proposition's *reporting* a fact is perhaps in need of clarification. Ordinarily, we would think of the reporting of a fact as a linguistic activity carried out by speakers. In those cases, I assume, the fact that a speaker reports a certain fact *f* in presenting as true a proposition *P* is due in part to a certain relevant relationship between *P* and *f*, for which I also use the word 'report'. I assume, moreover, that we may extend the range of that relation to all propositions, independently of whether they are, or indeed can be, expressed by a speaker of some given language.

[10] In the linguistic literature on questions, a distinction is standardly made between strongly exhaustive and weakly exhaustive answers, which mirrors the distinction adverted to in the text. Thus, a strongly exhaustive answer to the question who came to the party must also specify who did not come, whereas a weakly exhaustive one need only say who did come.

[11] Unlike the existence of negative truths, the existence of negative facts is controversial. The complaint is therefore less obviously compelling as applied to the second, vertical proposal.

to entail every truth about what there is, and what it is like, not the truths about what there is *not*, and what things are *not* like (cf. (Leuenberger, 2014: pp. 529f)).

Fortunately, it appears that there is a systematic way of bridging the gap between a complete truth in this weaker sense and a complete truth in the stronger sense. For if P is a complete truth in the weaker sense, then firstly, it is true to say that P holds, *and that's it*. Secondly, this strengthened truth appears then to entail all the missing negative truths. For instance, if we add to the complete physical description of the world the claim that *that's it*, the result does seem to rule out the existence of demons. This suggests that we can accommodate the objection against the above horizontal and vertical proposals by extending it with an account of the *totality operator* 'that's it'.[12] I shall take up this task in section 6 below. Until then, I shall focus on the stronger notion of completeness.

There is a second way in which the suggested characterizations of the notion of a complete truth may seem too demanding, namely in that they require a complete truth to be maximally specific: to describe its subject matter in full detail. But returning to Jack's breakfast, there is a truth and a fact pertaining to that subject matter stating *exactly* how much orange juice Jack had. But clearly, Jack's account of his breakfast does not have to entail that truth, or report that fact, in order to meet the contextually relevant standard of completeness. Strictly speaking, therefore, our account of the whole truth should be relativized to a contextually relevant level of specificity. Since making such relativization explicit would mainly be a distraction in the discussion to follow, I shall leave it implicit throughout, and merely occasionally comment on how the notion of a level of specificity could be implemented formally.

For definiteness, let me state explicitly the two informal characterizations of that notion suggested above:

(Horizontal): A proposition P is a complete truth (wrt subject matter m) iff (i) P is true, and (ii) P entails every truth (pertaining to m).

(Vertical): A proposition P is a complete truth (wrt subject matter m) iff (i) P is true, and (ii) P reports every fact (pertaining to m).

The two characterizations are neither obviously equivalent nor obviously incompatible. They jointly entail that a truth entails every truth (pertaining to a given subject matter m) iff it states every fact (pertaining to m), and relative to that assumption, they are equivalent. Pending clarification of the notion of entailment and the notion of fact-stating, it

[12] A suggestion like this is made in Chalmers and Jackson (2001), and formally elaborated in Leuenberger (2014). The linguistic accounts of questions and (exhaustive) answers sometimes make use of an 'exhaustivization' operator that plays a very similar role. An operator of this sort was first introduced by (Groenendijk and Stokhof, 1984: esp. ch. V).

is unclear whether the assumption should be taken to hold. It seems plausible, however, that the notions of entailment and fact-stating should permit of reasonably natural explications under which the assumption comes out true. And given the independent intuitive appeal of (Horizontal) and (Vertical), it seems desirable to give such an explication of the characterizations.

In order to develop our preliminary informal characterizations into an adequate and precise analysis of the notion of a complete truth, we thus need to clarify the key concepts invoked in (Horizontal) and (Vertical) in an appropriate way. More specifically, we need to come up with appropriate answers to the following questions:[13]

Q.1: What is a proposition?
Q.2: What is it for a proposition to be true?
Q.3: What is it for a proposition to entail another?
Q.4: What is a state of affairs?
Q.5: What is it for a proposition to report a state of affairs?
Q.6: What is it for a state of affairs to obtain, i.e. to be a fact?

Moreover, at least if we wish to give an explicit treatment of subject matter restrictions, we also need to answer these additional questions:

Q.7: What is a subject matter?
Q.8: What is it for a proposition to pertain to a subject matter?
Q.9: What is it for a state of affairs to pertain to a subject matter?

So let us see how this might be done.

3. Against intensional analyses

We may formulate prima facie natural and attractive answers to the above questions by appeal to the notion of a possible world, which for present purposes we may take as primitive. We may then answer Q.1–Q.3 above in the familiar way:

PW.1: A proposition is a set of possible worlds.
PW.2: A proposition is true iff it has the actual world as a member.
PW.3: A proposition P entails a proposition Q iff P is a subset of Q.

[13] Admittedly, since (Horizontal) and (Vertical) invoke the relations of entailment and reporting only with respect to truths and facts, respectively, the questions below are slightly more general than is perhaps strictly required for the purpose of clarifying these characterizations. But since we are naturally interested not just in which propositions actually are complete truths, but also in which propositions would be complete truths under various counterfactual circumstances, we shall eventually have to answer the more general questions anyway.

Let us set aside for the moment the matter of subject matter restrictions. Given PW.1–PW.3, we obtain the following account of the unrestrictedly whole truth:

PW.UWT: A proposition P is an unrestrictedly complete truth iff $P = \{@\}$.

(@ is the actual world). To see this, note that since $\{@\}$ is a truth, any complete truth must entail, and hence be a subset of, $\{@\}$. Since the empty set is a false proposition, $\{@\}$ itself is the only true proposition satisfying this constraint.

The most obvious way to capture a notion of a state (of affairs[14]) within the possible worlds framework is to identify it with the proposition that the state obtains, i.e. the set of worlds in which the state obtains. For a state to obtain is then simply for the proposition it is identified with to be true. Q.4–Q.6 may then be answered as follows:

PW.4: A state of affairs is a proposition.
PW.5: A proposition P reports a state of affairs s iff P entails s.
PW.6: A state of affairs obtains iff it is true.

Evidently, with entailment and state-reporting thus explicated, both (Horizontal) and (Vertical), in their unrestricted versions, come out true under the present approach.

I accept that PW.UWT captures one reasonable sense of 'complete truth', and I accept that there may be natural applications of the notion of a complete truth for which PW.UWT is plausible. But, I maintain, there are also many applications of that notion with respect to which PW.UWT is wholly implausible. The easiest way to see this is to observe that in many contexts in which we may evaluate truths for completeness, necessary truths, and necessary consequences, do not come for free.[15] Let me give two examples.

(1) Plausibly, all of the truths of pure set theory are necessary. So they are all identified, under the possible worlds conception, with the set of all possible worlds. So, under the conception of entailment captured in PW.3, *every* proposition, and hence every truth, entails all of them. Hence, the truth that the empty set has no members, for example, is classified as a complete truth with respect to the subject matter of set theory. But this seems absurd. At least, there would appear to be a good sense of 'complete truth' in which this is not so.

(2) Plausibly, Socrates has his nature non-contingently: if Socrates is essentially F, then in every world in which Socrates exists, Socrates is essentially F. But then under

[14] For brevity, I shall henceforth often simply speak of states.

[15] This kind of difficulty is of course familiar from many other applications of the possible worlds framework. It is nevertheless worthwhile to go through the issue in some detail, partly because it will help us identify the source of the difficulty in this application, and because it will provide us with a useful foil in the construction of a more satisfactory account below.

the possible worlds approach, it is implausibly easy to give a complete account of the nature of Socrates. It suffices to state, firstly, that Socrates exists, and secondly, Socrates is essentially F only if the truth that Socrates exists entails that Socrates is essentially F. Again, this seems absurd. Certainly, there seems to be a good sense of 'complete truth' in which this is not so.[16]

Strictly speaking, both examples concern the notion of a *restrictedly* complete truth. But it is clear that this is an inessential feature of the examples. The restriction to subject matter plays no role in how the difficulty arises, it only makes it simpler to specify concrete examples of truths incorrectly classified as complete. Still, it is worth nevertheless to briefly examine how PW.UWT may be extended to cover restricted notions of completeness. Following Lewis (1988b,a), we may take a subject matter m to be (represented by) an equivalence relation $\sim_m$ on the set of all possible worlds. The idea is that two worlds stand in the equivalence relation $\sim_m$ to one another just in case they are exactly alike as far as the subject matter m is concerned.

We then need to say what it is for a proposition to *pertain* to a subject matter. For a given equivalence relation $\sim_m$ on the set of worlds W and a world $w \in W$, let $[w]_m = \{v \in W : v \sim_m w\}$ be the equivalence class of w under $\sim_m$. Now consider a proposition P, and suppose that $w \in P$. We may then ask whether P is true at *every* world v for which $w \sim_m v$, i.e. at every world which is exactly like w with respect to m. If not, then we may conclude that P is *not just about m*, but says something about how things are that do not concern the subject matter m. If so, it should not be a requirement on a complete truth with respect to m that it entail P. So we may take a proposition P to pertain to a subject matter m iff $[w]_m \subseteq P$ whenever $w \in P$. (This is equivalent to how Lewis characterizes a proposition's being *entirely about* a subject matter, cf. (Lewis, 1988a: p. 163).[17]) For explicitness, we state the resulting answers to Q.7–Q.9:

PW.7: A subject matter m is an equivalence relation $\sim_m$ on the worlds.

PW.8: A proposition P pertains to a subject matter m iff for all $w \in P$, $[w]_m \subseteq P$.

PW.9: A fact pertains to a subject matter iff the proposition that the fact obtains pertains to that subject matter.

We then obtain the following account of a restrictedly complete truth:

[16] Note that the metaphysical assumptions about essence are inessential. We might simply imagine someone raising the question which truths involving Socrates are necessary given that Socrates exists. Then under the intensional analysis, the answer 'Socrates exists' would come out as expressing the whole truth about that subject matter.

[17] The same formal machinery of equivalence relations on the set of worlds could also be used to represent levels of specificity. An equivalence relation on the worlds would then be taken to represent a standard for specificity or level of detail on which exactly the non-equivalent worlds are distinguished.

PW.RWT: A proposition P is a complete truth wrt m iff $\{@\} \subseteq P \subseteq [@]_m$.

Applying this apparatus to the example (1), we see that the subject matter of pure set theory ends up being identified with the universal relation on the worlds, since all worlds are alike with respect to pure sets. So there are only two propositions pertaining to that subject matter, namely the impossible proposition that is true at no worlds, and the necessary proposition true at all worlds. As a result, any truth comes out as a complete truth with respect to the subject matter of pure set theory. In the case of (2), the subject matter of the nature of Socrates is represented by a relation in which two worlds stand just in case either both contain Socrates or neither contains Socrates. The strongest truth pertaining to that subject matter is then the set of worlds in which Socrates exists, i.e. the proposition that Socrates exists. Hence a truth is complete with respect to the subject matter of the nature of Socrates iff it entails that Socrates exists.

Where has the possible worlds analysis gone wrong? It seems to me that the explication of state-reporting that the analysis involves is very implausible. In particular, it seems to me that the true proposition that Socrates exists does not, intuitively, (also) report the state that Socrates is human—even though necessarily, the state obtains if the proposition is true. Likewise, it seems to me that the true proposition that no empty set has members does not, intuitively, (also) report the state that the empty set is a member of its singleton—even though necessarily, the state obtains if the proposition is true. What is missing, in spite of the presence of the right sort of *modal* connection between truth and state, is an appropriate connection of *relevance*.

For a proposition P to report a state, I would like to suggest, requires that the state be *wholly relevant* to making P true. And the state that Socrates is human is not wholly relevant to making it true that Socrates exists. It may not be wholly *irrelevant*, admittedly. For clearly, the state that Socrates exists is wholly relevant to making it true that Socrates exists, and it is not implausible to suppose that the state that Socrates is human contains the state that Socrates exists as a part. But then the state that Socrates is human still contains *more* than just the state that Socrates exists, and what it contains beyond this state is intuitively irrelevant to making it true that Socrates exists. Similarly, the state that the empty set is a member of its singleton is not wholly relevant for making it true that no empty set has members. Here it even seems that the state is *wholly irrelevant* to the truth of that proposition.

This diagnosis suggests that a number of otherwise tempting strategies for refining the possible worlds analysis are non-starters. In particular, one might have been tempted to respond to the above difficulties by saying that a truth is complete with respect to a subject matter only if it *logically* entails every truth pertaining to that subject matter. But as is well known, even (classical) logical entailment does not guarantee a connection of relevance between a proposition and what it entails. Still, it is worthwhile to confirm the point by briefly discussing how the proposal might be implemented. One way to do this is to work with a more relaxed conception of a possible world that covers even metaphysically impossible worlds, so long as they are still logically possible.[18] One might then take there to be worlds without any sets, and worlds in which Socrates exists but is not human, since these are not logically inconsistent. But it seems to me that even logical truths and consequences should not in general come for free. For instance, a complete truth about identity should report the fact that if $x = y$ and $y = z$ then $x = z$. For a truth to report this fact, the fact needs to be relevant to the truth. And the fact it is not relevant to every truth. It is entirely irrelevant, for example, to the truth that snow is white. So a proposition should not automatically count as reporting the fact that identity is transitive, just because that fact obtains as a matter of logical necessity.

4. A hyperintensional, truthmaker analysis

The discussion of the previous section suggests that in order to obtain an adequate analysis of complete truths, we need to take proper account of when a state is, or is not, *wholly relevant* to making a given proposition true. So what is it for a state to be thus relevant to a proposition? We may distinguish between two approaches to this question, the definitional and the axiomatic one. Under the definitional approach, we start by trying to analyse the pertinent notion of relevance in different, independently understood terms, and then see how the relation thus defined behaves, and whether it fits our intuitive demands. Under the axiomatic approach, we begin by taking some connection of relevance between states and propositions as basic, and develop its theory as best we can in accordance with our intuitive understanding of the notion.[19] I shall here pursue the latter approach. The basic relevantist connection I shall avail myself of is that of *wholly relevant verification*, which is the key concept underlying the truthmaker conception of content as recently developed by Kit Fine.[20]

[18] If one does not wish to do this, one would have to consider sentences rather than propositions as bearers of completeness. This strikes me as unattractive in any case, since intuitively it is the content of a sentence, not the sentence itself, which is evaluated for completeness. But independently of that, the proposal would face the same problem as the one discussed in the main text.

[19] Once we have done that, we may of course, in a second and optional step, proceed to examine whether we might be able to give a reductive definition of our relevance connection under which it behaves as required by our theory.

[20] See esp. Fine (2017a,b). The presentation of truthmaker semantics to follow is heavily indebted to these works.

Within the truthmaker conception, the usual appeal to a category of possible worlds is replaced by an appeal to a more general category of *states*. The states are assumed to be ordered by a relation of part-whole ($\sqsubseteq$), and it is assumed that given any set of states $T = \{s_1, s_2, \ldots\}$, we may form the mereological fusion $\bigsqcup T = s_1 \sqcup s_2 \sqcup \ldots$ defined as the smallest state of which all of the fused states are parts. In contrast to a world, a state may be *incomplete*: it may leave open the truth-value of many propositions. It may also be *inconsistent* or *impossible*: it may verify both of a pair of incompatible propositions.[21] A notion of a possible world may be recovered as the notion of a maximal consistent state, i.e. a state that contains every state it is compatible with. We shall assume that every consistent state in the state-space is part of a possible world. An obtaining state may then be identified with a *fact*, and the fusion of all facts with *the actual world* @. A proposition P is true iff verified by at least one fact, or equivalently, if @ contains some verifier of P.

A proposition P is identified with a pair (P^+, P^-) of a non-empty set P^+, comprising the verifiers of P, and a non-empty set P^-, comprising the falsifiers of P. Note that there is no requirement that if a given state s verifies P, then any state t containing s as a part must verify P as well; this reflects the intuitive requirement that verification be *wholly* relevant. We do, however, impose modal constraints on how the verifiers and the falsifiers may be related. In particular, we demand that no falsifier be compatible with any verifier—call this *Exclusivity*—and that for every proposition, every world contains either a verifier or a falsifier of that proposition—call this *Exhaustivity*.[22]

Operations of conjunction, disjunction and negation on the propositions may then be defined as follows:

$(\neg P)^+ \quad = P^-$

$(\neg P)^- \quad = P^+$

$(P \wedge Q)^+ \quad = \{s \sqcup t : s \in P^+ \text{ and } t \in Q^+\}$

[21] It should be made clear that in the formal development of the theory, the distinction between possible and impossible states is officially taken as primitive. Two states are then said to be incompatible iff their fusion is impossible, and two propositions are incompatible iff every verifier of the one is incompatible with every verifier of the other. So when I here characterize an impossible state as one that verifies both of a pair of incompatible propositions, this should be taken as merely an informal gloss, relying on an intuitive understanding of what it means for a pair of propositions to be incompatible. Thanks here to an anonymous referee.

[22] Intuitively, these constraints ensure bivalence of every proposition at every world, i.e. that at every world every proposition is either true or false but not both. Say that the propositions $P_1, P_2, \ldots$ modally entail Q iff Q is true at every world at which all of $P_1, P_2, \ldots$ are true. It can be shown that the logic of modal entailment for propositions satisfying *Exclusivity* and *Exhaustivity* is classical. (See Fine (2017a) for discussion, from where I have also borrowed the terminology.)

$(P \wedge Q)^- \quad = P^- \cup Q^-$
$(P \vee Q)^+ \quad = P^+ \cup Q^+$
$(P \vee Q)^- \quad = \{s \sqcup t : s \in P^- \text{ and } t \in Q^-\}$

We are now ready to answer some of the questions Q.1–Q.9:[23]

TM.1: A proposition is a pair of a set of verifiers and a set of falsifiers.
TM.2: A proposition is true iff at least one of its verifiers obtains.
TM.4: A state is anything that can play the role of truthmaker for a proposition.
TM.6: A state of affairs obtains iff it is part of the actual world.

Let us consider next how the notion of state-reporting may best be explicated within this framework. In order to ensure that a state be wholly relevant to the truth of any proposition reporting it, it suffices to demand that the state be part of some verifier of the proposition. In order to also ensure that if a proposition reports a state, the truth of the proposition entails that the state obtains, we demand that the state be part of *every* verifier of the proposition.

TM.5: A proposition P reports a state s iff s is part of every verifier of P.

In principle, a weaker condition would have sufficed to ensure that the truth of a proposition requires the obtaining of any fact it states, namely that every verifier of the proposition *necessitates* the fact in question.[24] However, this would lead us to count the proposition that Socrates exists or (Socrates is human and Socrates is not human) as reporting the fact that Socrates is human. For that fact is part of the impossible state that Socrates is both human and not human, which verifies the second disjunct, and the fact is moreover necessitated by every verifier of the first disjunct. This seems to be an undesirable result. The non-modal, more wholeheartedly relevantist explication TM.4 provides a more natural and theoretically well-behaved conception of state-reporting.

Focusing for the moment on unrestrictedly whole truths, we now have all the material in place required for a full truthmaker-theoretic interpretation of (Vertical): a proposition P is a complete truth iff (a) P is true, so P is verified by some fact and (b) P reports every fact, so every verifier of P contains every fact. Condition (a) is equivalent to the

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23 Note that TM.4 is offered simply as a potentially helpful gloss on what counts as a state. Officially, we are taking the notion of a state as basic, much as we did for the notion of a possible world in the previous section.

24 If we assume that every consistent state is part of a maximal consistent state, i.e. a world, we may say that a state s necessitates a state t iff every world that contains s contains t. More generally, a state s necessitates a state t iff every state that is compatible with s is compatible with t.

condition that some verifier of P be part of @, and (b) to the condition that @ be part of every verifier of P.[25]

TM.UWT: P is an unrestrictedly complete truth iff @ verifies P, and @ $\sqsubseteq s$ whenever s verifies P.

So under this approach, a truth is complete iff it reports the maximal fact that is the actual world, and thus only if the entire actual world is wholly relevant to the verification of the truth. A complete truth is allowed to have further verifiers, as long as these contain the actual world as a proper part. Note that since the actual world is a world, and hence a maximal consistent state, any such additional verifiers will be inconsistent states. One might thus consider singling out for special attention the 'pure' complete truths that are verified only by @. For now we rest content with TM.UWT.[26]

Let us now consider the notion of entailment and the interpretation of (Horizontal), as well as the question of its equivalence with (Vertical). It turns out that there is a very natural notion of entailment we can define within the truthmaker framework under which (Horizontal) comes out equivalent to (Vertical) as interpreted via TM.5. This is the relation that Fine calls *inexact entailment*, which obtains between a proposition P and a proposition Q just in case every verifier of P contains a verifier of Q as part (cf. e.g. (Fine, 2017a: p. 669)).[27]

TM.3: A proposition P entails a proposition Q iff every verifier of P contains a verifier of Q.

For suppose that P is a whole truth as per (Vertical). Then P is verified by @, and @ $\sqsubseteq s$ whenever s verifies P. We may then show that P inexactly entails every truth, and so is

25 Since @ is a fact, if every verifier of P contains every fact, every verifier of P contains @. Since every fact is part of @, if every verifier of P contains @, by transitivity of part, every verifier of P contains every fact.—Given that every set of states T, including the empty set, has a fusion $\bigsqcup T$, which is the least upper bound of the set with respect to $\sqsubseteq$, we may dually also take any set of states T, including the empty set, to its biggest common part $\bigsqcap T$, which is the greatest lower bound of T. Writing P^+ for the set of verifiers of P, our condition might therefore be expressed more succinctly as: @ $\in P^+$, and $\bigsqcap P^+ =$ @.

26 Note that even insisting on purity in the sense described will not yield uniqueness, since there will be many propositions that are verified only by @ but differ with respect to their falsifiers.

27 Strictly speaking, in the passage mentioned, Fine uses the term 'inexact consequence' and defines it for sentences, and by a slightly different condition which is however easily seen to be equivalent to the one I use. The differences are immaterial for our purposes.—It bears emphasis that my claim is simply that in order to obtain a plausible version of (Horizontal), the notion of entailment invoked in this principle should be understood as per TM.3. I do not intend to make any claim about what the 'real' notion of entailment is. In particular, I do not deny that every case of classical logical entailment or of ordinary modal entailment is a bona fide case of entailment.

a whole truth as per (Horizontal). For let Q be any truth. Then @ contains some verifier of Q, call it t. Since every verifier of P contains @ as a part, and @ contains t as a part, it follows that every verifier of P contains t as a part. So P inexactly entails Q, and since Q was arbitrary, P inexactly entails every truth, and hence is a whole truth in the sense of (Horizontal). For the other direction, suppose P is a whole truth as per (Horizontal), so P is true and inexactly entails every truth. Then in particular, P inexactly entails every truth that is verified solely by @. So all of P's verifiers must contain @. Since P is true, at least one of P's verifiers is a fact. Since @ is the only fact that contains @, it follows that one of P's verifiers is @. So P reports every fact and hence is a whole truth as per (Vertical).

It is relatively straightforward to extend this account to accommodate notions of completeness restricted to some subject matter.[28] A subject matter may be thought of in the first instance as the question which of a certain set of states obtain. For example, the subject matter of the colour of my shirt may be thought of as the question which states to the effect that my shirt is of a certain colour obtain. Plausibly, if for each state s_i of some states $s_1, s_2, \dots$ it pertains to a given subject matter m whether s_i obtains, then it also pertains to m whether $s_1 \sqcup s_2 \sqcup \dots$ obtains, and it pertains to m whether a state t obtains whenever $t \sqsubseteq s_1 \sqcup s_2 \sqcup \dots$ Given this assumption, we may simply represent a subject matter by a single state, namely the fusion of all states such that it pertains to the subject matter whether the state obtains.[29]

A state will therefore be taken to pertain to a subject matter iff it is a part of that subject matter. When should we take a proposition to pertain to a subject matter? I propose to take this to require that all the verifiers and all the falsifiers of the proposition be parts of the subject matter. For suppose a proposition P has a verifier (falsifier) that is not part of a given subject matter m. Then there is a way for P to be true (false) which turns on matters foreign to m. This seems to constitute a good sense in which P does not pertain purely or wholly to m. The proposal may also be inferred from Fine's suggestion (cf. (Fine, 2017b: p. 676ff)) that the subject matter of a proposition may be identified with the fusion of all its verifiers and all its falsifiers, given the plausible assumption that

[28] For a more detailed discussion of the notion of subject matter within truthmaker semantics, see (Fine, 2017b: pp. 676ff).

[29] Note that a subject matter will typically be an impossible state, since it will result from fusing many mutually incompatible states. For instance, since both the state of my shirt being red all over and the state of my shirt being blue all over pertain to the subject matter of the colour of my shirt, the state representing it will contain both as part and therefore be impossible.—One might consider imposing a number of constraints on a state if it is to qualify as a subject matter, but we shall not discuss the matter here.

a proposition's own subject matter should be the smallest subject matter the proposition wholly pertains to. Finally, the claim that this understanding of pertaining to a subject matter is appropriate at least for our purposes receives further confirmation from the fact that it allows us to establish the desired equivalence of (Horizontal) and (Vertical).

First, we make explicit our answers to Q.7–Q.9

TM.7: A subject matter is any state.

TM.8: A state pertains to a subject matter iff it is part of the subject matter.

TM.9: A proposition pertains to a subject matter iff it is verified and falsified only by parts of the subject matter.

A restricted counterpart to TM.UWT may then be stated as follows:

TM.RWT: P is a complete truth with respect to subject matter m iff every verifier of P contains $@ \sqcap m$, and some verifier of P is part of $@$.

(Here $@ \sqcap m$ denotes the biggest state that is part of both $@$ and m, and hence the biggest fact pertaining to m.)

Under the interpretation provided by this account, (Horizontal) and (Vertical) are equivalent. For assume that P is a complete truth with respect to subject matter m according to (Horizontal). That is, P is true and P inexactly entails every truth pertaining to m. Then in particular, P inexactly entails every truth pertaining to m that is verified only by $@ \sqcap m$. So all of P's verifiers contain $@ \sqcap m$. Hence P is true and reports every fact pertaining to m, and so is a complete truth with respect to m according to (Vertical). For the converse direction, assume that P is true and reports every fact pertaining to m. Let Q be a truth pertaining to m. Then every verifier of Q is part of m, and since Q is true, at least one of them is also part of $@$, and hence part of $@ \sqcap m$. But then since P reports every fact pertaining to m, P reports $@ \sqcap m$, hence every verifier of P contains $@ \sqcap m$ and thus, by the transitivity of part, the mentioned verifier of P.[30]

This truthmaker-based analysis of the notion of a complete truth avoids the difficulties discussed above for the possible worlds analysis.[31] Case (1), recall, was that of pure set

[30] We noted in the context of the intensional account of the whole truth that the formal machinery used to represent subject matters—i.e. equivalence relations on the set of worlds—could also be used to represent levels of specificity. Under the truthmaker account, we are representing subject matters in a very different way, which will not be of help in capturing levels of specificity. Instead, we can do something similar to what we did in the possible worlds framework by taking the *congruence relations* on a state-space to represent levels of specificity. Roughly, these are *order-preserving* equivalence relations on the state-space.

[31] It is worth pointing out that the analysis does not make essential use of the most distinctive aspect of exact truthmaker semantics, i.e. the exactness requirement on verification: to determine whether a proposition is a whole truth (with respect to some subject matter), it is enough to know what its *inexact*

theory. The problem was that under the possible worlds analysis, every truth comes out as complete truth with respect to the subject matter of pure set theory, since every truth pertaining to that subject matter is necessary, and thereby vacuously entailed by every proposition under the modal construal of entailment. In our diagnosis of the problem, we argued that the state s that the empty set is a member of its singleton is not wholly relevant for making true the proposition P that no empty set has members. So s is an actual part of the subject matter m of pure set theory, so $s \sqsubseteq @ \sqcap m$. But P does not report s, for s is not part of every, or indeed any verifier of P, since it is not wholly relevant to the truth of P. Hence P is true, and even a truth pertaining to the subject matter of set theory, but under the truthmaker approach it is not a complete truth with respect to that subject matter.

Case (2) was about the subject matter of the nature of Socrates, and the problem was that every truth that entails that Socrates exists is automatically classified by the possible worlds analysis as a complete truth with respect to that subject matter. In our diagnosis of the problem, we argued that the state that Socrates is human is not wholly relevant to making it true that Socrates exists. So the truth that Socrates exists does not report the fact that Socrates is human, and since that is a fact pertaining to the subject matter, that truth is correctly classified by the truthmaker approach as not a complete truth with respect to the nature of Socrates.

5. Do we still get too much for free?

The problem for the possible worlds analysis was that it gave away necessary truths and consequences for free, as it were, and that in many contexts it seemed that a putative complete truth should not get them for free. As we saw, the truthmaker analysis does not face the same difficulty. Just because a proposition is necessarily true, or a necessary consequence of a given putative complete truth P, it does not follow that it is inexactly entailed by P, and hence the truthmaker analysis does not give it to P for free. This is fine as far as it goes, one might respond, but it does not mean that there is not a narrower class of truths or consequences that even our account gives away for free, and for which contexts may be found in which a putative complete truth should not get these for free. This section is devoted to answering this concern.

The truthmaker analysis does indeed give away certain things for free. Just as the possible worlds analysis gives away for free all the necessary consequences of a putative

verifiers are, which are exactly those states that contain an exact verifier as part. As we shall see below, though, in order to obtain a satisfactory account of the totality operator 'that's it', and thereby of the weaker sense of completeness described in section 2, we need to appeal to exact truthmaking. Thanks to an anonymous referee for raising this matter.

complete truth, so the truthmaker analysis gives away for free any *inexact* consequences of a putative complete truth *P*. And just like the possible worlds analysis gives away for free all the necessary truths, there may also be a special sort of truth that the truthmaker analysis gives away for free. Recall that for any set of states, we may form their fusion. It is normally assumed in the context of the truthmaker framework that this holds even for the empty set of states. The result of fusing no states, as it were, is called the *nullstate*, which is the one state that is part of absolutely every state. Now say that a proposition is *trivial* iff it is verified by (perhaps among other things) the nullstate.[32] Then every proposition inexactly entails every trivial proposition. Hence any putative complete truth is automatically classified as entailing, in the relevant sense, any trivial proposition, and likewise it is automatically classified as reporting the nullstate.

Let me describe a few classes of examples that might at first glance appear problematic for my account. Firstly, there is the class of *conceptual entailments*. Plausibly, that Kant was a bachelor inexactly entails that Kant was unmarried. For presumably, the state of Kant's having been a bachelor is the same state as the state of Kant's having been an unmarried male, which surely contains as a part the state of Kant's having been unmarried. Now, while this particular example of a conceptual entailment is fairly obvious, there may well be conceptual, inexact entailments that are not obvious. In such cases, one might suspect that the failure to make explicit a relevant truth conceptually entailed by a truth *P* might well intuitively disqualify *P* from completeness.

Secondly, there are cases of *grounding*. At least under the semantics for grounding described in Fine (2012b), Fine (2012a), and Fine (2017b), any case of (full) grounding is a case of inexact entailment. But truths of grounding can be highly non-obvious, for instance if the truths about a person's mental life are indeed fully grounded in physical truths. And presumably one can ask questions about a person's mental life such that no answer consisting purely of physical truths would intuitively be considered acceptably complete.

Thirdly, there may be *trivial logical truths*. Of course, most logical truths are not trivial under the truthmaker treatment. Indeed, it is easy to see that there is no way to form a trivial truth by application of the usual truth-functional operations to propositions which are neither trivially true nor trivially false. For when the application of any such operation brings new verifiers or falsifiers into play, these are always states properly containing states that were already in play as verifiers or falsifiers of the propositions

[32] Note that this is a not an epistemic notion of triviality; there is no obvious and reason for thinking that every proposition that is verified by the nullstate should be immediately recognizable as true by anyone who entertains it.

to which the operation is applied.[33] There may, however, be other, reasonably natural operations on propositions that do allow this. The most plausible candidate I am aware of is the *incremental conditional* $P \to Q$ of (Fine, 2014: esp. §4).[34] One obtains a verifier of $P \to Q$ by first considering any function that maps every verifier s of P to a verifier of Q. For s verifying P, one then considers the smallest state t such that $s \sqcup t$ contains $f(s)$. Finally, one takes the fusion of all these smallest states. In the case of $P \to P$, taking the identity function that maps every verifier of P to itself then yields the nullstate as verifier of $P \to P$, so $P \to P$ is trivially true. But this might appear problematic. For to the extent that sometimes it is not admissible to omit $\forall x\ x = x$ from a complete truth, perhaps it is sometimes also not admissible to omit an instance of $P \to P$, and so a putative complete truth should not get that instance for free.

Finally, there may be *trivial non-factive grounding truths*. On the currently best developed account of iterated grounding claims, due to Litland (2017), true non-factive grounding claims are *zero-grounded*: grounded, but by the empty collection of facts or truths.[35] Still assuming a treatment of ground within truthmaker semantics along the lines described by Fine, a truth will turn out zero-grounded iff verified by the nullstate.[36] And surely there are contexts in which truths of non-factive grounding may not simply be omitted from a truth without sacrificing its completeness.

A minimal response one might give to this objection would be as follows: Yes, there are cases in which trivial truths or consequences do not come for free, but the truthmaker approach gives these away for free, and so in these cases the approach does not give the right results. But at least it can handle a lot more cases than its possible-worlds based rival, and so it still constitutes a step in the right direction, even if it does not quite get us where we would ideally want to be.

However, I think that a less concessive response is warranted. True, there are contexts in which trivial truths may not be omitted from a truth without rendering that truth incomplete in the contextually relevant sense. But it seems to me that that sense of completeness is different from the sense of completeness at issue in the cases I used

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33 Contrast this with the situation of the necessary truth in the possible worlds framework, which may be obtained from any proposition P by an application of negation and disjunction to form $P \vee \neg P$.

34 Note that in that paper, Fine does not call the conditional in question 'incremental'. I am borrowing this apt term from Yablo (2016), and Fine's exchange with Yablo on Yablo (2014); see Fine (ms) and Yablo (2018).

35 The idea that some truths might be grounded in this way is due to (Fine, 2012a: p. 47f).

36 In general, under the truthmaker semantical account, a truth P is (weakly fully) grounded by the collection of truths Γ just in case for every choice T of one verifier each from the members of Γ, the fusion $\bigsqcup T$ of the states in T verifies P (cf. (Fine, 2012a: § 1.10),(Fine, 2017b: pp. 685ff, 700ff)). In the case where Γ is empty, that means that $\bigsqcup \emptyset$ must verify P, and $\bigsqcup \emptyset$ is just the nullstate.

to motivate the truthmaker approach, and that it is different in *kind*, not just in *degree*. There is, on the one hand, a wide range of applications in which the pertinent notion of a complete truth is such that, intuitively, a truth's being complete is a matter of the truth describing both the *entirety* of the relevant portion of reality, and doing so *in full detail*—in other words, a matter of the truth reporting every fact that is part of the relevant portion of reality. It is to these applications that the present account is intended to apply. And in these applications, trivial truths and entailments do come for free, since they can neither help report further parts of reality, nor can they add to the level of detail in which a portion of reality is being described.

In the contexts in which trivial truths and entailments do not come for free, completeness is not, or not purely, a matter of which parts of reality are being described and at what level of detail or specificity. Rather, in these contexts, completeness (also) demands that we answer, roughly speaking, for each of a relevant range of *representations* of reality, whether they represent reality accurately. To meet this demand, we may have to employ several accurate representations of the same parts of reality. So here we have a partly representational notion of completeness, whereas in the other cases we have what may call a purely *worldly* notion of completeness. And it would be a mistake to try and capture these different notions within a single theory; they should be treated separately.[37]

With respect to the above examples, this response commits me to a disjunction: either the examples are not actually cases of inexact entailment, or they invoke a notion of a complete truth outside the scope of my account, targeting representational rather than purely worldly completeness. Which of the disjuncts obtains is, in at least some of the cases, a difficult question. For what it is worth, I incline towards the second disjunct with respect to the conceptual entailments and trivial logical truths. I suspect there might be more to be said for the first disjunct with respect to the grounding cases. But either way, the disjunction can be seen to be generally plausible irrespective of which disjunct one favours in each case. For once one has taken a view on the truthmakers of the relevant propositions under which the entailments in question are genuine inexact entailments, one has already taken a view under which the propositions merely provide different representations of the same portions of reality.

[37] A similar distinction between a worldly and a representational version of a notion is often made with respect to grounding. Here, too, a truthmaker based approach may be seen as adequate for the worldly, though perhaps not the representational notion; cf. e.g. Correia (2010, 2017); Fine (2017b); Krämer (2018).

6. Totality operators

We return, finally, to the issue of totality operators and their connection to the notion of a complete truth. In order for Jack to give a complete answer to the question what he had for breakfast, recall, he need not mention that he did not eat a crocodile. He may just say that he had eggs, bacon, coffee, *and that's it*. Indeed, the final totality clause would normally be taken as understood. Similarly, at least in one important sense, the complete physical description of the world may be the whole truth about the world, even though it does not entail that there are no demons. It is sufficient if the result of appending 'and that's it' to the complete physical description of the world entails that truth.

Given the central role that totality operators thus seem to play in the formulation of many complete truths, a theory of the notion of a complete truth would appear incomplete if it lacked an account of these operators. While it is beyond the scope of this paper to give a fully developed account, I shall attempt in this section to describe the broad outlines of a truthmaker-based treatment of totality operators, and to highlight a few advantages such a treatment seems to me to offer in comparison to possible-worlds based rivals.

§6.1 offers an account of the unrestricted totality operator. §6.2 briefly describes how the account may be extended to accommodate restricted totality operators. §6.3 considers an alternative possible-worlds based rival first proposed in Chalmers and Jackson (2001) and developed further by Leuenberger (2014), and argues that the present account is superior.

6.1. **An Unrestricted Totality Operator.** Our task is to define a one-place operation Δ on propositions that corresponds to the intuitive understanding of relevant uses of 'that's it', and that bridges the gap between a truth that is complete in the weaker sense exemplified by Jack's description of his breakfast and a truth that is complete in the stronger sense of inexactly entailing every truth pertaining to the relevant subject matter. Within the truthmaker framework, that task divides into two parts, that of specifying the verifiers and that of specifying the falsifiers of ΔP. We begin with the verifiers.

The basic idea is to take there to be a function δ on the states, such that for any given state s, the state δs is the state of s obtaining, *and that's it*. Assuming that there is a coherent notion of an unrestricted *that's it* in application to propositions, it is very plausible that there should be such a function. To see this, note that for any given state s, we may consider a proposition P verified only by s. It seems very plausible that ΔP should then also have just a single verifier. For if there is only one way for P to be true, it is hard to see how there could be several ways for it to be the case that P is true, and that's it. But then we may simply let δs be the state verifying ΔP, where P is verified only by s.[38]

In giving an account of the δ function, we may again pursue either a definitional or an axiomatic approach. Just as I did for the relation of relevant verification before, I shall here adopt the axiomatic approach. So I shall merely lay down some intuitively plausible principles connecting the δ function to the mereological and modal aspects of the state-space, draw out some of their consequences, but leave open the question of the definability of δ in other, independently understood terms.

We may start with three somewhat rough, intuitive ideas about the behaviour of δ that are suggested by its informal reading in terms of 'that's it'. First, since δs is the state to the effect that *s obtains*, and that's it, δs should contain s as a part. Second, since δs is the state to the effect that s obtains, *and that's it*, in some sense, δs should not contain anything more than s. Third, δs should be incompatible with any state it does not contain.

It is easy to see that, were it not for the hedge 'in some sense' in the second principle, the three principles would not be jointly satisfiable except in special cases, namely when s is already incompatible with any state it does not contain. The first and third principle are straightforward and will be taken for granted henceforth:

δ.1: $s \sqsubseteq \delta s$
δ.2: δs is incompatible with any state it does not contain

With respect to the second principle, we need to ask how it may be clarified. A natural thought is that δs should not be allowed to contain anything *positive* beyond what is contained in s, it should only be allowed to go beyond s in a purely *negative* way, namely by precisely ruling out the obtaining of any further positive facts not already contained in s. Let us take for granted, for the moment, an exclusive and exhaustive distinction among the states between wholly positive states and partly negative states. It is very plausible to suppose that (a) a state is partly negative whenever it has a partly negative part, and (b) that any fusion of wholly positive states is itself wholly positive. We may then define the positive part s^p of a state s as the fusion of all its wholly positive parts. In this terminology, the idea that δs may only go beyond s in a purely negative way may be captured as the claim that s and δs always have the same positive part. In addition, it seems plausible that the output of the δ-function should (a) depend only to

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38 Since there may be several propositions P verified solely by s but differing with respect to their falsifiers, I am also relying here on the assumption that the verifier of ΔP does not vary with the falsifiers in these cases. Again, this seems intuitively highly plausible.

the positive part of its input, and (b) differ whenever the positive parts of the input states differ. We thus arrive at two further plausible constraints on δ:

δ.3**:** $(\delta s)^p = s^p$
δ.4**:** $\delta s = \delta t$ iff $s^p = t^p$

We now turn to the question how the verifiers of ΔP may be obtained from δ and the verifiers of P when P has more than just one verifier. In effect, this is the question of how Δ should be taken to behave in application to a *disjunction*. For a proposition with multiple verifiers $s_1, s_2, \ldots$ is essentially the same as the disjunction of the propositions $P_1, P_2, \ldots$ verified, respectively, only by $s_1, s_2, \ldots$ From a formal point of view, the most natural option would be to take ΔP to be verified by δs whenever s verifies P, and by no other states. That option also seems to fit well with our intuitive judgements. To elicit clear intuitions about the matter, it is best to consider again some examples with an implicitly restricted subject matter. A fairly natural example of an application of 'that's it' to a disjunction is as follows. Suppose we are asking Jack what he has had to drink for lunch—suspecting, perhaps, that he's been on the booze. He might reply: I had two or three glasses of orange juice—I don't quite remember—and that's it! Then the most natural way of interpreting him would be as saying, in effect, that either he had two glasses of orange juice, and that's it, or he had three glasses of orange juice, and that's it. This is just as predicted by the suggested account, which I therefore propose we accept:

$(\Delta P)^+ \quad = \{\delta s : s \text{ verifies } P\}$

It should be noted, though, that this makes it relatively easy for ΔP to come out true. All one needs to do is formulate a truth that, intuitively, has a sufficiently large subject matter that the entire actual world, or at least its positive part, is relevant to verifying the truth. And since relevant verification does not, on the truthmaker conception, imply any sort of *minimality*, this may be relatively easy. For instance, if we give the positive part of the world a name, say 'Bill', then it will presumably be true that Bill exists, and that's it. Indeed, under a natural account of the truthmakers of existential quantifications, it will be true that something exists, and that's it. While this is perhaps a somewhat odd or anomalous result, I don't think it is too objectionable, or even too counter-intuitive. A statement such as that something exists, and that's it, intuitively strikes one as odd and unhelpful rather than clearly false, it seems to me—and odd and unhelpful it is even under the present account. It is worth emphasizing, moreover, that the truth of ΔP is not sufficient for P to be classified as a *whole truth*. That ΔP is true guarantees only that @ verifies ΔP, whereas in order for P to be a whole truth, *every* verifier of ΔP must contain the actual world. This further condition is of course not met in the cases just mentioned.

We turn now to the question of the falsifiers of ΔP. There are three main constraints on how it may be answered. First, in order for to falsify ΔP, a state must be incompatible with every verifier of ΔP, otherwise it would be possible for ΔP to be both true and false. Second, to guarantee that in every possible world, ΔP is either true or false, we need to make sure that every world contains a falsifier of ΔP if it does not contain a verifier of ΔP. Third, a falsifier of ΔP should be wholly relevant to making ΔP false. Unfortunately, it is not easy to come up with an answer that satisfies all three constraints. We might start by considering the maximally liberal account of the falsifiers of ΔP on which every state incompatible with every verifier of ΔP is considered a falsifier of ΔP.

$$(\Delta P)^- \quad = \{s : s \text{ is incompatible with } \delta t \text{ whenever } t \text{ verifies } P\}$$

This answer evidently satisfies the first constraint, and it is also easily shown to satisfy the second constraint.[39] The problem is the third constraint, demanding that a falsifier be wholly relevant. Generally speaking, a purely modal condition like the one of incompatibility is not sufficient to guarantee a relevance connection.

Actually, it turns out that the tension of the proposal with the relevance constraint is less dramatic than one might expect. Consider first the case in which all of the verifiers of ΔP are consistent, and hence possible worlds. Then for a state to be incompatible with each of them is equivalent to its not being a part of any of them. And it seems quite plausible that any such state is relevant to making it false that ΔP. Somewhat metaphorically, we might think of any state as saying of itself that it is a fact, and of a state δs as additionally saying of itself that it contains every fact. Thus, any state t not contained in δs presents itself, wholly relevantly, as a counter-example to the claim we take δs to make.[40]

Unfortunately, this defence of the liberal account does not extend to the case in which all verifiers of ΔP are inconsistent and hence every state falsifies ΔP. The perhaps most counterintuitive kind of case we obtain is when a consistent state s falsifies such a proposition ΔP, while s is also part of every verifier of ΔP, and intuitively has nothing to do with the inconsistency of those verifiers. Thus, suppose we are considering the subject matter of the properties of a ball, and P is the proposition that the ball is red all over, green all over, and round. Then the state of the ball being round is a falsifier of

[39] Suppose w is a world which does not contain a verifier of ΔP. Since w is a world, it is incompatible with every state it does not contain, and so it is incompatible with every verifier of ΔP.

[40] In light of this, one might be tempted to say that we should take the falsifiers of ΔP to be simply those states that are not contained in any verifiers of ΔP. But the problem with this suggestion is that ΔP may be false and yet there may not be any states not contained in any verifiers of ΔP, for instance when ΔP is verified by the fusion of all states. So under this approach, we would be violating the second of the above constraints on the set of falsifiers of ΔP.

ΔP even though intuitively, it plays no part in making it false that ΔP. So it cannot be claimed that the liberal account fully captures the intuitive understanding of 'that's it'.

We are left with three options. The first is to try to somehow refine the liberal account to rule out the irrelevant falsifiers by appeal to their modal and mereological properties. The second is to assume more structure in the state-space. Specifically, we might take as given a relation of *wholly relevant exclusion* on the states and construct the falsifiers of ΔP by choosing a relevant excluder for each verifier of ΔP and building their fusion.[41] Both these options introduce a significant degree of additional complexity into the theory. A third option might therefore be to endorse the liberal account, and propose Δ, so interpreted, merely as a reasonably close approximation to the intuitive interpretation of 'that's it'. If the best version of the first two options turn out to yield a significantly more complex and messy theory, there may be good pragmatic justification for the third option.

6.2. **Restricted Totality Operators.** To accommodate totality operators restricted to a particular subject matter, we may work with a binary δ function that takes a subject matter as its second argument, so that $\delta(s, m)$ is the state to the effect that *as far as m is concerned*, s obtains, and that's it. The unrestricted, unary δ function may then be defined in terms of the binary one by setting m to the maximal subject matter, i.e. the fusion of all states. In developing this approach further, there are a number of choices to be made, and it is not always obvious in advance of detailed theorizing which of them is best. My aim here is simply to sketch the outlines of one natural approach we might take. While it may not ultimately be the best one, it provides some grounds for thinking that there is nothing in principle standing in the way of developing, within the overall framework here described, a satisfactory theory of restricted totality operators.

We shall assume that binary $\delta(s, m)$ will only be defined when $s \sqsubseteq m$, i.e. when s pertains to m. We may then lay down the following counterparts to (δ.1)–(δ.3) for $s \sqsubseteq m$:

(δ_R.1): $s \sqsubseteq \delta(s, m)$
(δ_R.2): $\delta(s, m)$ is incompatible with any part of m it does not contain
(δ_R.3): $\delta(s, m)^p = s^p$

These straightforwardly entail (δ.1)–(δ.3) under the proposed definition of unary δ.

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[41] Given the relation of relevant exclusion, we might then move to a unilateral conception of propositions as given simply by a set of verifiers, and characterize negation in general in the way that we just proposed to treat the negation of Δ-propositions. This alternative, exclusionary treatment of negation is discussed in more detail in (Fine, 2017a: pp. 634f, 658ff).

It is not as obvious what we should say about the conditions for the identity $\delta(s, m) = \delta(t, n)$ to hold. It is very plausible that the identity should only hold if $s^p = t^p$, since whatever the subject matter, δ is not allowed to add anything positive to the state it presents as total. It is also very plausible that if $s^p = t^p$ and $m = n$, then the identity should hold. But there may also be cases in which the identity holds even though $m \neq n$. Thus let s be the state that the ball is red, m the fusion of the ball's being red and the ball's being blue, and n the fusion of m and the ball's being green. Then it seems plausible that $s = \delta(s, m)$, since s is already incompatible with every part of m it does not contain. But likewise it would seem, for the same reason, that $s = \delta(s, n)$, even though $m \neq n$.

Now it may be felt that m is not a good candidate for a subject matter in an intuitive sense. For the only subject matter it could represent, one might think is that of the colour of the ball, and that subject matter seems better represented by n. But it is not immediately obvious how one might characterize those states that represent genuine subject matters, and even if this can be done, it may be desirable to allow $\delta(s, m)$ to be defined even when m is not a subject matter in this narrower sense. If so, then we may still impose the separate necessary and sufficient conditions for the identity of restricted totality-states just proposed:

(δ_R.4a): $\delta(s, m) = \delta(t, n)$ if $n = m$ and $s^p = t^p$
(δ_R.4b): $\delta(s, m) = \delta(t, n)$ only if $s^p = t^p$

Note that (δ_R.4a) implies (δ.4) under the proposed definition of unrestricted δ.

In addition to defining unary δ in terms of binary δ in the way indicated, we may also be able to define the notion of a partly negative state in terms of binary δ, and thereby further reduce the number of primitives assumed in our theory. In informal terms, the idea is to assume that totality-states are the only source of negativity, so that a state is partly negative if and only if it contains a totality-state. However, we need to be careful when making this precise. Since under the present account, every state s equals $\delta(s, s)$, we must not interpret the notion of a totality-state invoked in the informal idea as simply that of a value of δ for some arguments. Instead, let us call a state s a *proper totality-state* iff for some states $t \sqsubseteq m$, $s = \delta(t, m)$ and $t \neq s$. A proper totality-state can therefore be obtained by adding *that's it* to some *other*, strictly smaller state, and it must therefore be considered partly negative. We may then consider defining the set of partly negative states as the set of states containing some proper totality-state as a part.

If the definition is to work as intended, we need to make sure that the set of partly negative states, so defined, satisfies the two assumptions we made above. That is, we need to make sure that any state that contains a partly negative state is itself partly negative, and that any fusion of some wholly positive states is itself wholly positive. The first assumption is a trivial consequence of the proposed definition. The second is not, so we lay down as a further constraint on δ that

(δ_R.5): $s \sqcup t \sqcup \ldots$ contains a proper totality-state only if one of s, t, $\ldots$ does

Let us finally define a subject-matter relative totality operation on propositions in terms of binary δ. Just as we took $\delta(s, m)$ to be defined only when s pertains to m, we shall define $\Delta(P, m)$ only when P pertains to m, i.e. when the fusion of all verifiers and all falsifiers of P is a part of m.[42] The obvious adaptation of the previous verifier-clause, and liberal falsifiers-clause, to the restricted case is then as follows:

$$(\Delta(P, m))^+ = \{\delta(s, m) : s \text{ verifies } P\}$$
$$(\Delta(P, m))^- = \{s \sqsubseteq m : s \text{ is incompatible with } \delta(t, m) \text{ whenever } t \text{ verifies } P\}$$

The falsifier-clause raises the same worries that its unrestricted counterpart does, but as far as I can see it does not raise any new ones.[43]

6.3. **Comparison with Intensional Approaches.** It is instructive to compare the present, truthmaker-based account of Δ with an alternative, possible-worlds based approach that was first proposed by David Chalmers and Frank Jackson ((Chalmers and Jackson, 2001: p. 317f)) and then developed in formal detail by Stephan Leuenberger (Leuenberger (2014)). In the possible worlds framework, we need to say at which worlds ΔP is true, given the information at which worlds P is true. The proposal that Chalmers and Jackson make is that a world w verifies ΔP iff (a) w is a P-world, and (b) among the P-worlds, w is *minimal* with respect the relation of *outstripping*, which is a roughly parthood-like relation among the worlds. More precisely, a world w is said to outstrip another world v just in case w contains as a part an intrinsic *duplicate* of v, but v does not contain a duplicate of w. The informal idea is simple and clear enough: in order for a world w to make true ΔP, w must make true P, and w must be 'as small as possible' consistent with making P true, so that if we were to remove any of w's parts, the result would no longer make P true.

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42 Note that $\Delta(P, \blacksquare)$ will then always be defined, and we may again define the unary, unrestricted Δ in terms of the binary version by setting the subject matter to the fusion of all states.

43 The falsifier-clause illustrates the importance of requiring that m contain all the falsifiers as well as all the verifiers of P. Since any proposition is required to have a non-empty set of falsifiers, we need to ensure that m always contains some state incompatible with $\delta(t, m)$ whenever t verifies P if the clause is to be acceptable. Since any falsifier of P is incompatible with every verifier of P, and hence with $\delta(t, m)$ whenever t verifies P, the requirement that m contain the falsifiers of P ensures this, but absent this requirement, we would have no such guarantee.

Our own account conforms to a very similar idea: in order for a state s to verify ΔP, s must make P true in the sense of *containing* a verifier of P, and the entire positive part of s must be relevant to verifying P. Here the appeal to the positive part of a state serves a similar purpose as the appeal to duplication in Chalmers and Jackson. Chalmers and Jackson appear to think of worlds on a broadly Lewisian model as a spatio-temporally maximal concrete entity. A world, so construed, makes true a negative truth such as that there are no unicorns firstly, by not containing unicorns, and secondly, by being maximal, and therefore not part of a bigger entity that might include unicorns. Since a world is maximal, it cannot itself be a part of another world, only an intrinsic duplicate of one. In the truthmaker framework, worlds are seen as maximally consistent states of affairs rather than maximal spatio-temporal entities. A world w, so construed, makes true a negative truths such as that there are no unicorns by containing a partly negative part that is wholly relevant to there not being a unicorn. These parts of w of course need not be wholly relevant to verifying P in order that w verify ΔP, hence the need for the restriction to the positive part of w. We can use this observation to define a relation of outstripping on the worlds in the truthmaker framework, by saying that a world w outstrips a world v iff w^p properly contains v^p. It is then straightforward to show, given our constraints on the δ-function, that whenever δs is consistent, δs is the minimal element with respect to outstripping among all the worlds that contain s.[44]

From the truthmaker perspective, the crucial difference between the Chalmers-Jackson account and our own is at which the stage the minimality condition is applied. Let $s, t, \ldots$ be the verifiers of P, and assume for ease of comparison that $\delta s, \delta t, \ldots$ are all consistent and hence possible worlds. To obtain the worlds at which ΔP is true under the Chalmers-Jackson account, we first form, for each state among $s, t, \ldots$ the set of worlds containing that state, then build the union of these sets of worlds to obtain the set of worlds at which P is true, and finally apply the minimality condition to select the minimal elements among these. To obtain the worlds at which ΔP is true under our own account, we first form, for each state among $s, t, \ldots$ the set of worlds containing that state, then apply the minimality condition to obtain the minimal elements of each such set, and finally take the union of all the resulting (singleton) sets.[45]

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[44] More generally, δs is always the unique minimal element with respect to outstripping among all the δ-states that contain s as a part. For suppose that $s \sqsubseteq \delta t$. Then $s^p \sqsubseteq (\delta t)^p$, and since by ($\delta$.3), $(\delta s)^p = s^p$, we have $(\delta s)^p \sqsubseteq (\delta t)^p$. If $(\delta s)^p \sqsubset (\delta t)^p$ then δt outstrips δs. If not, then $(\delta s)^p = (\delta t)^p$, and we may infer by ($\delta$.3) that $s^p = t^p$, and by (δ.4) that $\delta s = \delta t$.

[45] Note that under a possible worlds approach, we have only the information at which worlds P is true to start with, so there is only one stage at which the minimality condition can be applied. It is because the truthmaker approach also takes into account which parts of the P-worlds are wholly relevant to P,

It seems to me that the second approach yields a more intuitive view of the truth-conditions of ΔP. Consider again the question of Jack's breakfast, and suppose that Jack had eggs, bacon, and nothing else. Suppose, however, that he gives the following answer: I had eggs, I had bacon, and I either had coffee or I didn't have coffee, and that's it. Then the intuitive account of his statement would seem to be as follows. What Jack said is strue, but it is *not* a strongly whole truth. For it does not entail the truth that Jack did not have coffee. Indeed, it seems to *explicitly* leave open the possibility that he did have coffee. The truthmaker approach is completely in agreement with this intuitive assessment. Let us abbreviate the sentences to which Jack's 'that's it' is applied as $(E \wedge B) \wedge (C \vee \neg C)$. The exact verifiers of that sentence include both a state s verifying $(E \wedge B) \wedge C$ and a state t verifying $(E \wedge B) \wedge \neg C$. As a result, the exact verifiers of $\Delta((E \wedge B) \wedge (C \vee \neg C))$ thus include δs—a minimal world in which Jack had eggs, bacon, and coffee—and δt—a minimal world in which Jack had eggs, bacon, and no coffee. Hence $\Delta((E \wedge B) \wedge (C \vee \neg C))$ does not inexactly or even modally entail the truth that Jack did not have coffee, and hence is not classified as a strongly complete truth.

But suppose Jack had said instead: I had eggs and bacon, and that's it. The intuitive assessment would then be different. Given that he did indeed have eggs, bacon, and nothing else, that statement would intuitively count as a strongly complete truth, as it would entail (among other things) that Jack did not have coffee. Again, this is the result that we get under the truthmaker approach. For none of the exact verifiers of the embedded statement $E \wedge B$ say anything about coffee. Suppose s is such a verifier. Then applying δ to s will yield a minimal world in which Jack had eggs and bacon, and hence a world in which Jack did not have coffee. After all, any state c to the effect that he did have coffee would be a positive state, and since no such state is part of s^p, by our constraint (δ.3), no such state is part of $(\delta s)^p$ either.

But since the two embedded statements $E \wedge B$ and $(E \wedge B) \wedge (C \vee \neg C)$ are logically equivalent, they are true at the same worlds. Under any possible-worlds approach, the content of ΔP can depend only on which worlds P is true at, and hence the statements $\Delta((E \wedge B) \wedge (C \vee \neg C))$ and $\Delta(E \wedge B)$ will in turn be true at exactly the same worlds. So in contrast to the truthmaker approach, no possible-worlds approach can respect the intuitive difference between the two statements. Instead, the Chalmers-Jackson approach counts $\Delta((E \wedge B) \wedge (C \vee \neg C))$ a strongly complete truth, since no world in which Jack had coffee is minimal among the worlds at which $(E \wedge B) \wedge (C \vee \neg C)$ is true. So just as the hyperintensional, truthmaker based account of the notion of a complete truth was

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i.e. which states are exact truthmakers of P, that it becomes possible to apply the minimality condition at a different and, as I shall argue, more appropriate stage.

seen above to be superior to intensional, possible-worlds based ones, so the hyperintensional, truthmaker based semantics for the totality operator turns out to be superior to intensional, possible-worlds based ones.[46]

It might be objected that the intuitive difference between $\Delta((E \wedge B) \wedge (C \vee \neg C))$ and $\Delta(E \wedge B)$ is perhaps a purely pragmatic difference, not a semantic one, and thus that it is only the content that is communicated, and not the content semantically expressed, that is compatible with Jack's having had coffee. Although I know of no decisive objection to that view, it does not strike me as particularly plausible. If we imagine Jack attempting to cancel the putative implicature, for example by adding 'By saying this, I do not mean leave it open that I had coffee', then this would seem to me to simply warrant the response: 'Well, maybe you did not mean to, but leave it open you did!' However, for the purposes of this paper, I can allow that there may be an alternative way of accommodating the intuitive data with respect to this example. The truthmaker approach is motivated in the first instance by the considerations of section 3 to the effect that necessary truths and consequences often do not come for free. And it still speaks in favour of the approach that it can handle the present examples naturally and elegantly, without the need to appeal to pragmatic interference, even if the account it offers is not obviously the only possible account.

7. Conclusions

We are typically interested not just in giving true descriptions of a subject matter, but in giving descriptions that are true and complete: the whole truth. In this paper, I have criticized extant, intensional accounts of the relevant notion of completeness and tried to develop a better, hyperintensional one, utilizing the framework of truthmaker semantics. According to the view I propose, a truth is complete with respect to a given subject matter if and only if every fact pertaining to the subject matter is wholly relevant to making the truth true.

In this strong sense, a truth is complete only if it (relevantly) entails even the negative truths concerning the subject matter in question. In many contexts, it seems appropriate

[46] It should be noted that while the notion of a strongly complete truth, as pointed out before, is sensitive only to differences in inexact verifiers, the totality operators and thereby the notion of a weakly complete truth is sensitive even to difference concerning only exact verifiers. Suppose Jack had said that he had eggs and bacon, or eggs and bacon and coffee, and that's it—formally, $\Delta((E \wedge B) \vee ((E \wedge B) \wedge C))$. Then his statement is verified both by a minimal world in which Jack had eggs and bacon, and by a minimal world in which Jack had eggs, bacon, and coffee. Intuitively, this is the right result. For as before, Jack's statement explicitly leaves open the possibility that he had coffee, whereas the statement that he had eggs and bacon, and that's it, does not.

to apply a weaker standard, under which only positive truths have to be implied by the truth in question. The gap between weakly and strongly complete truths may be bridged by means of the totality operator 'that's it'. A truth is weakly complete just in case the result of applying 'and that's it' to the truth yields a strongly complete truth. I have sketched a truthmaker semantics both for an absolute and a subject-matter relative totality operator. While it remains conceptually close in some ways to a previous account due to Chalmers and Jackson, we saw that the move to a hyperintensional framework once more allows in a very natural way to better capture the intuitive understanding of the operator. It remains to work out the semantics in full detail and to determine the logic of the totality operator. But that is a task I have to leave for another occasion.

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